THE PREP

St. Joseph's Preparatory School

HONORS ALGEBRA
Summer Review Packet

IMPORTANT NOTICE:

The following packet contains some basic pre-Algebra concepts that you should have mastered before arriving at school. Please take the time to read through the examples, complete all exercises and check your solutions. The first two days of class will be spent answering questions on the packet. Good luck. We look forward to meeting you in September.

A TEST ON THIS MATERIAL WILL BE GIVEN DURING THE FIRST CYCLE OF CLASSES.

TOPICS:

(1) Fractions
(2) Adding and Subtracting Positive and Negative Numbers
(3) Multiplying and Dividing Positive and Negative Numbers
(4) Absolute Value
(5) Order of Operations
(6) Solving Two-Step Equations
(7) Exponents
(8) Slope-Intercept Form

TOPIC: Fractions

Fraction - a mathematical expression that indicated a quotient of two quantities

Example: $\frac{2}{5}$ (2: numerator; 5: denominator)

N.B. Unless you are answering a real-world problem you should avoid using mixed numbers.

Improper Fraction - a fraction with a numerator that is larger than the denominator

Example: $\frac{7}{3}$

How to convert a mixed number into an improper fraction:

Step 1. Take the product (multiply) of the integer and the denominator.
Step 2. Add the product from Step 1 to the numerator.
Step 3. Denominator is the same.

Example: $3\frac{1}{5} = \frac{3\times 5+1}{5} = \frac{16}{5}$

OR

$$3\frac{1}{5} = 3+\frac{1}{5} = 1+1+1+\frac{1}{5} = \frac{5}{5}+\frac{5}{5}+\frac{5}{5}+\frac{1}{5} = \frac{16}{5}$$

Exercises A:
Convert the following mixed numbers to improper fractions.

1. $3\frac{1}{7}$ 2. $2\frac{7}{9}$ 3. $5\frac{3}{8}$ 4. $33\frac{1}{3}$
5. $12\frac{1}{4}$ 6. $4\frac{13}{15}$ 7. $1\frac{2}{3}$ 8. $66\frac{2}{3}$

How to Reduce a Fraction:

Step 1. Factor the numerator.
Step 2. Factor the denominator.
Step 3. Cancel common factors.

Example: $\frac{24}{90} = \frac{2\cdot 2\cdot 2\cdot 3}{2\cdot 3\cdot 3\cdot 5} = \frac{2}{2}\cdot\frac{3}{3}\cdot\frac{2\cdot 2}{3\cdot 5} = 1\cdot 1\cdot\frac{2\cdot 2}{3\cdot 5} = \frac{4}{15}$

OR

$$\frac{24}{90} = \frac{6\cdot 4}{6\cdot 15} = \frac{6}{6}\cdot\frac{4}{15} = 1\cdot\frac{4}{15} = \frac{4}{15}$$

Exercises B:
Reduce the following fractions.

1. $\frac{18}{21}$ 2. $\frac{72}{48}$ 3. $\frac{104}{65}$ 4. $\frac{25}{45}$ 5. $\frac{112}{42}$
6. $\frac{63}{18}$ 7. $\frac{99}{36}$ 8. $\frac{1024}{48}$ 9. $\frac{9}{51}$ 10. $\frac{84}{35}$

How to Multiply Fractions:

Step 1. Multiply the numerators.
Step 2. Multiply the denominators.
Step 3. Reduce the fraction if necessary.

Example: $\frac{3}{4}\cdot\frac{5}{6} = \frac{3\cdot 5}{4\cdot 6} = \frac{15}{24} = \frac{3\cdot 5}{3\cdot 8} = \frac{5}{8}$

Special Case: Sometimes you will need to multiply a fraction and an integer.

Step 1. Convert the integer to a fraction ("integer over 1").
Step 2. Multiply the numerators.
Step 3. Multiply the denominators.
Step 4. Reduce the fraction if necessary.

Example: $16\cdot\frac{5}{12} = \frac{16}{1}\cdot\frac{5}{12} = \frac{80}{12} = \frac{4\cdot 20}{4\cdot 3} = \frac{20}{3}$

Exercises C:
Multiply.

1. $\frac{5}{6}\cdot\frac{7}{4}$ 2. $\frac{3}{8}\cdot\frac{16}{21}$ 3. $\frac{15}{6}\cdot\frac{1}{3}$ 4. $4\cdot\frac{1}{3}$ 5. $\frac{7}{4}\cdot\frac{3}{28}$

6. $\frac{2}{5} \cdot 8$ 7. $\frac{9}{6} \cdot 12$ 8. $\frac{1}{3} \cdot \frac{5}{7}$ 9. $\frac{9}{51} \cdot 17$ 10. $\frac{4}{5} \cdot \frac{135}{100}$

Reciprocal - the quotient of 1 and a number.

Example: The reciprocal of $\frac{5}{7}$ is $\frac{7}{5}$.

The reciprocal of 5 is $\frac{1}{5}$.

How to Divide Fractions:

Step 1. Multiply by the reciprocal of the second fraction.
Step 2. See steps for multiplying fractions.
Step 3. Reduce the fraction if necessary.

Example: $$\frac{2}{3} \div \frac{8}{5} = \frac{2}{3} \cdot \frac{5}{8} = \frac{10}{24} = \frac{5}{12}$$

Exercises D:
Divide.

1. $\frac{5}{6} \div \frac{7}{4}$ 2. $\frac{3}{8} \div \frac{16}{21}$ 3. $\frac{15}{6} \div \frac{1}{3}$ 4. $4 \div \frac{1}{3}$ 5. $\frac{7}{4} \div \frac{3}{28}$
6. $\frac{2}{5} \div 8$ 7. $\frac{9}{6} \div 12$ 8. $\frac{1}{3} \div \frac{5}{7}$ 9. $\frac{9}{51} \div 17$ 10. $\frac{4}{5} \div \frac{135}{100}$

How to Add Fractions:

Step 1. Express both fractions in terms of a common denominator.
Step 2. Add/Subtract numerators.
Step 3. Reduce the fraction if necessary.

Example with common denominator:

$$\frac{3}{4}+\frac{7}{4}=\frac{3+7}{4}=\frac{10}{4}=\frac{2\cdot 5}{2\cdot 2}=\frac{5}{2}$$

Example without common denominator:

$$\frac{1}{3}+\frac{5}{6}=\frac{2}{2}\cdot\frac{1}{3}+\frac{5}{6}=\frac{2}{6}+\frac{5}{6}=\frac{7}{6}$$

Example without common denominator:

$$\frac{7}{4}-\frac{1}{18}=\frac{9}{9}\cdot\frac{7}{4}-\frac{2}{2}\cdot\frac{1}{18}=\frac{63}{36}-\frac{2}{36}=\frac{61}{36}$$

Exercises E:
Simplify.

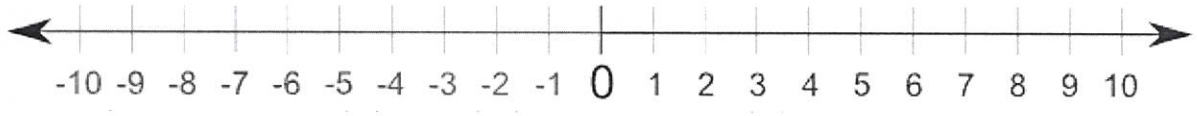
1. $\frac{3}{5}+\frac{7}{5}$
2. $\frac{5}{6}+\frac{10}{6}$
3. $\frac{2}{3}-\frac{1}{6}$
4. $\frac{6}{7}+\frac{5}{14}$
5. $\frac{3}{4}-\frac{3}{4}$
6. $\frac{1}{3}+\frac{3}{4}$
7. $\frac{5}{7}-\frac{2}{5}$
8. $\frac{5}{6}+\frac{18}{12}$
9. $\frac{1}{4}-\frac{1}{5}$
10. $\frac{7}{8}+\frac{5}{3}$

TOPIC: Adding and Subtracting Positive and Negative Numbers

Every number has a position on the Real Number Line.

Positive Numbers are to the **Right** of Zero.

Negative Numbers are to the **Left** of Zero.



Addition moves to the right.

Subtraction moves to the left.

Adding a Positive Number:

Adding positive numbers is just simple addition.

Example: 2 + 3 = 5

is really saying

"Positive 2 plus Positive 3 equals Positive 5"

You could write it as (+2) + (+3) = (+5)

Subtracting a Positive Number

Subtracting positive numbers is just simple subtraction.

Example: 6 - 3 = 3 **is really saying:**

"Positive 6 minus Positive 3 equals Positive 3"

You could write it as (+6) - (+3) = (+3)

Adding a Negative Number:

Adding a negative number is **SUBTRACTION.**

Example: 6 + (-2) = 6 - 2 = 4

Subtracting a Negative Number:

Subtracting a negative number is **ADDITION.**

Example: 7 - (-3) = 7 + 3 = 10

Rules:

Two "like" signs become positive (+)

Two "unlike" signs become negative (-)

Exercises F: Simplify.

1. $5 - 9$ 2. $8 + (-12)$ 3. $\frac{5}{6} - \frac{4}{3}$ 4. $\frac{1}{5} + \left(-\frac{8}{15}\right)$

5. $-8 - 11$ 6. $-\frac{2}{3} - \frac{2}{3}$ 7. $-\frac{4}{7} - 9$ 8. $-\frac{15}{8} + 4$

TOPIC: Multiplying and Dividing Positive and Negative Numbers

Rules:

The product (result of multiplication) or quotient (result of division) of "like" signs is positive (+).

The product (result of multiplication) or quotient (result of division) of "unlike" signs is positive (-).

Examples:

$$(-4)(-5) = +20$$

$$\left(-\frac{1}{5}\right)\cdot 15 = -\frac{15}{5} = -3$$

$$-8 \div 2 = -4$$

$$-16 \div \left(-\frac{4}{3}\right) = \frac{-16}{1}\cdot\frac{-3}{4} = \frac{48}{4} = 12$$

Exercises G:
Simplify.

1. $(-5)\cdot(-3)$

2. $-4\cdot 9$

3. $6\left(-\frac{5}{12}\right)$

4. $\left(-\frac{1}{3}\right)\left(-\frac{6}{5}\right)$

5. $(-18)\div(-6)$

6. $-9\div 12$

7. $\frac{1}{7}\div\frac{-8}{14}$

8. $\left(-\frac{5}{4}\right)\div\left(-\frac{21}{16}\right)$

TOPIC: Absolute Value

Absolute Value is the distance a number is from zero. Absolute value is always nonnegative.

$$|8| = 8$$

"The absolute value of 8" is the distance 8 is from zero on the real number line. 8 is 8 units from zero on the number line.

$$|-4| = 4$$

"The absolute value of -4" is the distance -4 is from zero on the real number line. -4 is 4 units from zero on the number line.

Exercises H:

1. $|9|$ 2. $|-14|$ 3. $|0|$ 4. $\left|-\frac{1}{5}\right|$ 5. $\left|\frac{12}{4}\right|$

TOPIC: Order of Operations

The order of operations is the sequence in which you must perform algebraic operations.

You have probably learned the mnemonic "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally" or PEMDAS where

P - parentheses (any grouping symbol such as brackets or a fraction bar)
E - exponents
M - multiplication
D - division
A - addition
S - subtraction

Since multiplication and division are complimentary operations as are addition and subtraction, you should remember it slightly differently.

P
E
M/D - Work from left to right when deciding whether to perform multiplication or division first.
A/S - Work from left to right when deciding whether to perform addition or subtraction first.

Example:

$7 + (6 \times 5^2 + 3)$

Step 1: Work inside the grouping symbol.

$7 + \left(6 \times 5^2 + 3\right)$

Step 2: Do the exponent.

$7 + (6 \times 5^2 + 3) = 7 + (6 \times 25 + 3)$

Step 3: Do the multiplication.

$7 + (6 \times 25 + 3) = 7 + (150 + 3)$

Step 4: Do the addition.

$7 + (150 + 3) = 7 + 153$

Step 5: Do the addition.

$7 + 153 = 160$

$7 + (6 \times 5^2 + 3)$
$= 7 + (6 \times 25 + 3)$
$= 7 + (150 + 3)$
$= 7 + (153)$
$= 160$

More Worked Examples:

$$10 - 5^3$$
$$= 10 - 125$$
$$= -115$$

$$10 \div 2 \cdot 5$$
$$= 5 \cdot 5$$
$$= 25$$

$$\frac{2 \cdot 7^2 - 90}{12}$$
$$= \frac{2 \cdot 49 - 90}{12}$$
$$= \frac{98 - 90}{12}$$
$$= \frac{8}{12}$$
$$= \frac{2}{3}$$

$$7 - 5\left|3 - 2^3\right|$$
$$= 7 - 5\left|3 - 8\right|$$
$$= 7 - 5\left|-5\right|$$
$$= 7 - 5 \cdot 5$$
$$= 7 - 25$$
$$= -18$$

Exercises I:
Simplify.

1. $5 - 4 \cdot 3^2$

2. $6 - 3(2 + 3)^2$

3. $12 - 7\left|5 - 2 \cdot 3^2\right|$

4. $\left(\frac{1}{3}\right)^2 \div \frac{2}{5} + \frac{1}{6}$

5. $2\left[(3 - 9)^2 + 5\right] - 5$

6. $-2^4 + 6\left|6 - 8\right|$

7. $\left(-\frac{6}{5}\right)^2 \div \frac{2}{15} + \frac{1}{5}$

8. $\left[2\left(3 - \frac{1}{4}\right)^0\right]^1$

9. $4\left(\frac{1}{5} + \frac{3}{2}\right) \div \left(\frac{15}{2} - 8\right)^2$

TOPIC: Solving Two-Step Equations

Solving Two-Step Equations

A couple of hints:

1. To solve an equation, UNDO the order of operations and work in the reverse order.
2. REMEMBER! Addition is "undone" by subtraction, and vice versa. Multiplication is "undone" by division, and vice versa.

Ex. 1: $4x - 2 = 30$

$+2 \quad +2$

$4x = 32$

$\div 4 \quad \div 4$

$x = 8$

Ex. 2: $87 = -11x + 21$

$-21 \qquad -21$

$66 = -11x$

$\div -11 \quad \div -11$

$-6 = x$

Solving Multi-step Equations With Variables on Both Sides of the Equal Sign

- When solving equations with variables on both sides of the equal sign, be sure to get all terms with variables on one side and all the terms without variables on the other side.

Ex. 3: $8x + 4 = 4x + 28$

$-4 \qquad -4$

$8x = 4x + 24$

$-4x \quad -4x$

$4x = 24$

$\div 4 \quad \div 4$

$x = 6$

Solving Equations that need to be simplified first

- In some equations, you will need to combine like terms and/or use the distributive property to simplify each side of the equation, and then begin to solve it.

Ex. 4: $5(4x - 7) = 8x + 45 + 2x$

$20x - 35 = 10x + 45$

$-10x \qquad -10x$

$10x - 35 = 45$

$+35 \quad +35$

$10x = 80$

$\div 10 \quad \div 10$

$x = 8$

Exercise J:

Solve each equation. You must show all work.

1. $5x - 2 = 33$

2. $140 = 4x + 36$

3. $8(3x - 4) = 196$

4. $45x - 720 + 15x = 60$

5. $132 = 4(12x - 9)$

6. $198 = 154 + 7x - 68$

7. $-131 = -5(3x - 8) + 6x$

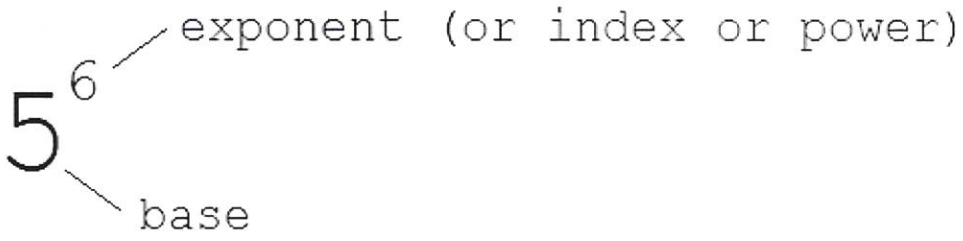
8. $-7x - 10 = 18 + 3x$

9. $12x + 8 - 15 = -2(3x - 82)$

10. $-(12x - 6) = 12x + 6$

TOPIC: Exponents

The **exponent** of a number indicates how many times the number should be multiplied.



$$5^6 = (5)(5)(5)(5)(5)(5) = 15625$$

Notes:

Anything raised to the 1^{st} power simply equals the base.
Anything raised to the 0^{th} power equals 1.
Positive numbers raised to any power are positive.
Negative numbers raised to odd powers are negative.
Negative numbers raised to even powers are positive.

Exercises K:
Simplify.

1. 3^4
2. 5^2
3. $(-8)^3$
4. $(-3)^2$
5. $(819)^1$
6. $(-54)^0$
7. $\left(\frac{3}{5}\right)^2$
8. $\left(-\frac{5}{2}\right)^3$

Rules of Exponents

Multiplication: Recall $(x^m)(x^n)=x^{(m+n)}$ *Ex:* $(3x^4y^2)(4xy^5)=(3\cdot 4)(x^4\cdot x^1)(y^2\cdot y^5)=12x^5y^7$

Division: Recall $\frac{x^m}{x^n}=x^{(m-n)}$ *Ex:* $\frac{42m^5j^2}{-3m^3j}=\left(\frac{42}{-3}\right)\left(\frac{m^5}{m^3}\right)\left(\frac{j^2}{j^1}\right)=-14m^2j$

Powers: Recall $(x^m)^n = x^{(m\cdot n)}$ *Ex:* $(-2a^3bc^4)^3 = (-2)^3(a^3)^3(b^1)^3(c^4)^3 = -8a^9b^3c^{12}$

Power of Zero: Recall $x^0 = 1,\ x \neq 0$ *Ex:* $5x^0y^4 = (5)(1)(y^4) = 5y^4$

Exercise L:

Simplify each expression.

1. $(c^5)(c)(c^2)$
2. $\frac{m^{15}}{m^3}$
3. $(k^4)^5$
4. d^0
5. $(p^4q^2)(p^7q^5)$
6. $\frac{45y^3z^{10}}{5y^3z}$
7. $(-t^7)^3$
8. $3f^3g^0$
9. $(4h^5k^3)(15k^2h^3)$
10. $\frac{12a^4b^6}{36ab^2c}$
11. $(3m^2n)^4$
12. $(12x^2y)^0$
13. $(-5a^2b)(2ab^2c)(-3b)$
14. $4x(2x^2y)^0$
15. $(3x^4y)(2y^2)^3$

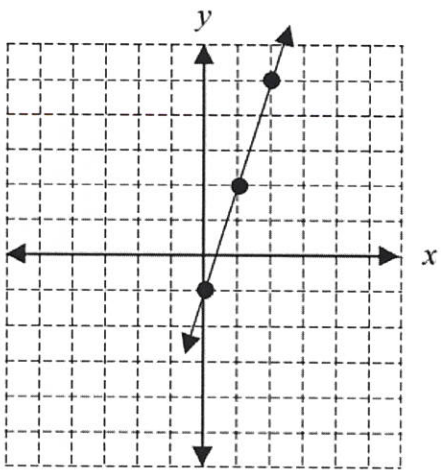
TOPIC: Slope-Intercept Form

Using the Slope – Intercept Form of the Equation of a Line.

The slope-intercept form for the equation of a line with slope m and y-intercept b is $y = mx + b$.

Ex. $y = 3x - 1$

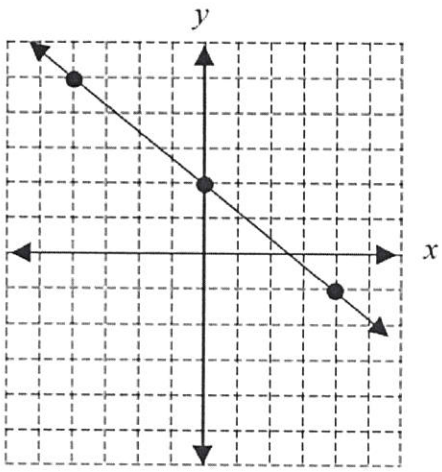
Slope: 3 y-intercept: -1



Place a point on the y-axis at -1. Slope is 3 or 3/1, so travel up 3 on the y-axis and over 1 to the right.

Ex. $y = -\frac{3}{4}x + 2$

Slope: $-\frac{3}{4}$ y-intercept: 2

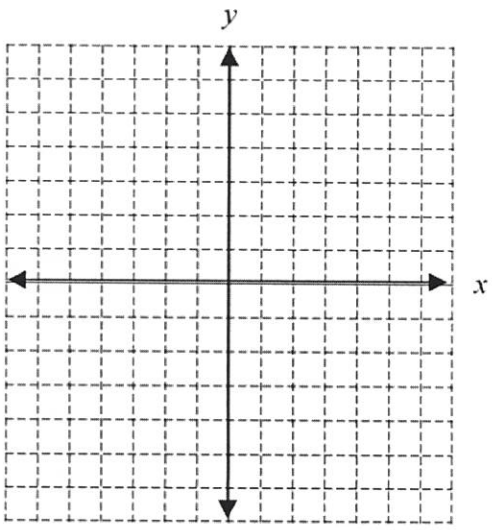


Place a point on the y-axis at 2. Slope is -3/4 so travel down 3 on the y-axis and over 4 to the right. Or travel up 3 on the y-axis and over 4 to the left.

Graph.

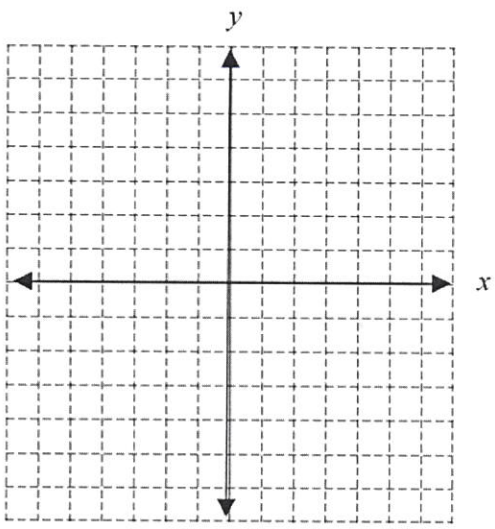
1. $y = 2x + 5$

Slope: _____ y-intercept: _____



2. $y = \frac{1}{2}x - 3$

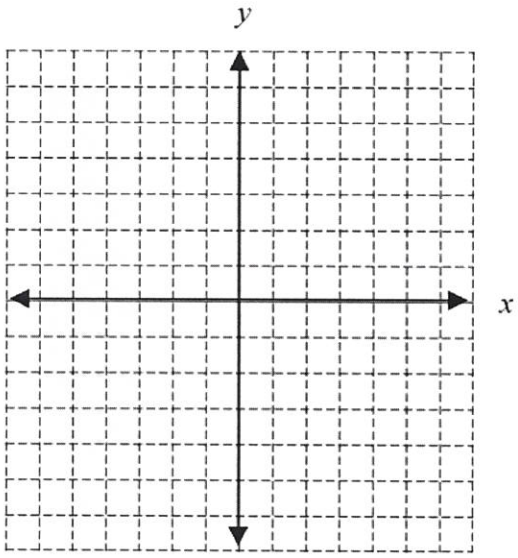
Slope: _____ y-intercept: _____



3. $y = -\frac{2}{5}x + 4$

Slope: _____

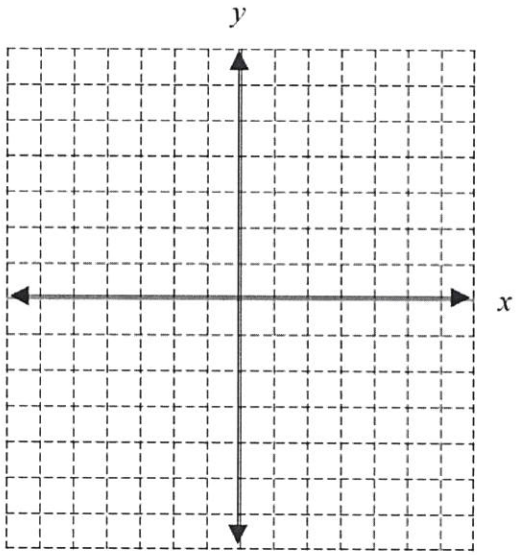
y-intercept: _____



4. $y = -3x$

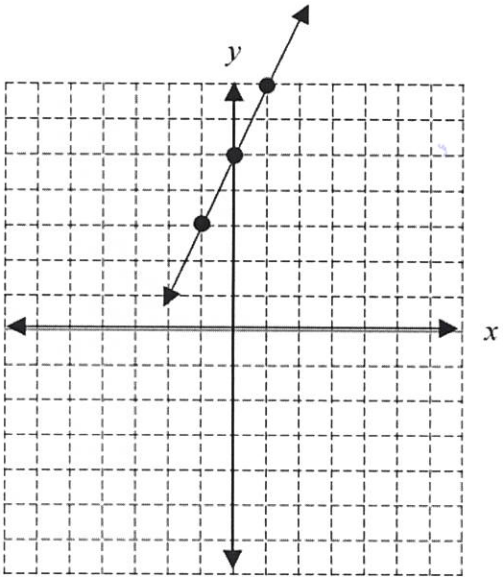
Slope: _____

y-intercept _____

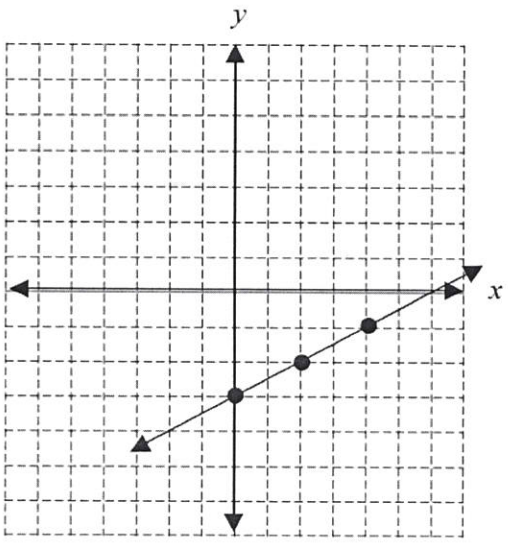


Graph.

1. Slope: **2** y-intercept: **5**



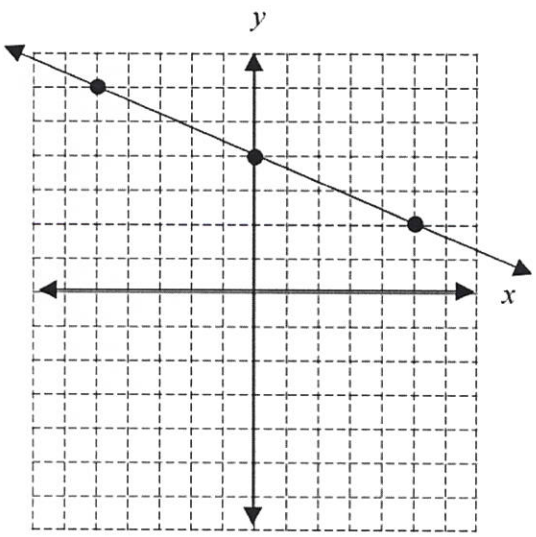
2. Slope: $\frac{1}{2}$ y-intercept: **-3**



3. $y = -\frac{2}{5}x + 4$

Slope: ____$-\frac{2}{5}$_____

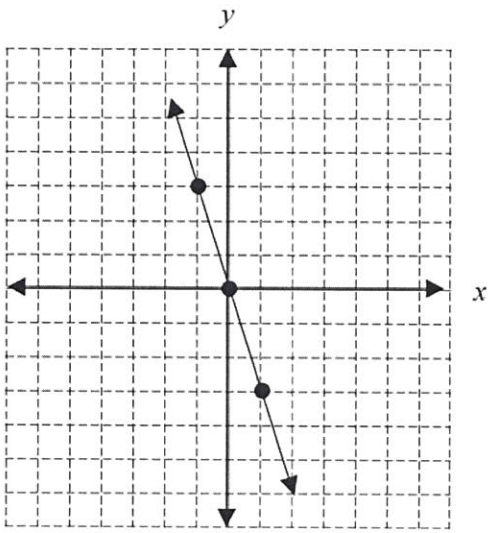
y-intercept: _____**4**_____



4. $y = -3x$

Slope: _____**-3**_____

y-intercept _____**0**_____



ANSWERS

Exercises A:

1. $\frac{22}{7}$, 2. $\frac{25}{9}$, 3. $\frac{43}{8}$, 4. $\frac{100}{3}$, 5. $\frac{49}{4}$, 6. $\frac{73}{15}$, 7. $\frac{5}{3}$, 8. $\frac{200}{3}$

Exercises B:

1. $\frac{6}{7}$, 2. $\frac{3}{2}$, 3. $\frac{8}{5}$, 4. $\frac{5}{9}$, 5. $\frac{8}{3}$, 6. $\frac{7}{2}$, 7. $\frac{11}{4}$, 8. $\frac{64}{3}$, 9. $\frac{3}{17}$, 10. $\frac{12}{5}$

Exercises C:

1. $\frac{35}{24}$, 2. $\frac{2}{7}$, 3. $\frac{5}{6}$, 4. $\frac{4}{3}$, 5. $\frac{3}{16}$, 6. $\frac{16}{5}$, 7. 18, 8. $\frac{5}{21}$, 9. 3, 10. $\frac{27}{25}$

Exercises D:

1. $\frac{10}{21}$, 2. $\frac{63}{128}$, 3. $\frac{15}{2}$, 4. 12, 5. $\frac{49}{3}$, 6. $\frac{1}{20}$, 7. $\frac{1}{8}$, 8. $\frac{7}{15}$, 9. $\frac{9}{867}$, 10. $\frac{16}{27}$

Exercises E:

1. 2, 2. $\frac{5}{3}$, 3. $\frac{1}{2}$, 4. $\frac{17}{14}$, 5. 0, 6. $\frac{13}{12}$, 7. $\frac{11}{35}$, 8. $\frac{7}{3}$, 9. $\frac{1}{20}$, 10. $\frac{61}{24}$

Exercises F:

1. −4, 2. −4, 3. $-\frac{1}{2}$, 4. $-\frac{1}{3}$, 5. −19, 6. $-\frac{4}{3}$, 7. $-\frac{67}{7}$, 8. $\frac{17}{8}$

Exercises G:

1. 15, 2. −36, 3. $-\frac{5}{2}$, 4. $\frac{2}{5}$, 5. 3, 6. $-\frac{3}{4}$, 7. $-\frac{1}{4}$, 8. $\frac{20}{21}$

Exercises H:

1. 9, 2. 14, 3. 0, 4. $\frac{1}{5}$, 5. 3

Exercise I:

1. −31, 2. −69, 3. −79, 4. $\frac{4}{9}$, 5. 77, 6. −4, 7. 11, 8. 2, 9. $\frac{136}{5}$

Exercise J:

1. $x = 7$
2. $x = 26$
3. $x = 9.5$
4. $x = 13$
5. $x = 3.5$
6. $x = 16$
7. $x = 19$
8. $x = -2.8$
9. $x = 9.5$
10. $x = 0$

Exercises K:

1. 81, 2. 25, 3. −512, 4. 9, 5. 819, 6. 1, 7. $\frac{9}{25}$, 8. $-\frac{125}{8}$

Exercises L:

1. c^8
2. m^{12}
3. k^{20}
4. 1
5. $p^{11}q^7$
6. $9z^9$
7. $-t^{21}$
8. $3f^3$
9. $60h^8k^5$
10. $\frac{a^3b^4}{3c}$
11. $81m^8n^4$
12. 1
13. $30a^3b^4c$
14. $4x$
15. $24x^4y^7$