St. Joseph's Prep

Department of Mathematics

Algebra II

Summer Review Packet

Instructions: Complete this review packet in its entirety. All of this content should be review. Be prepared to ask questions on this review packet the first day of classes. There will be a test on the first B day of the first quarter. You may use a calculator as well.

SOLVING EQUATIONS

EX1: $13y+48=8y-47$

$5y+48=-47$ subtract 8y from both sides

$5y=-95$ subtract 48 from both sides

$y=-19$ divide both sides by 5

EX 2: $3x-7(2x-13)=3(-2x+9)$

$3x-14x+91=-6x+27$ Use the distributive property

$-11x+91=-6x+27$ Combine like terms

$-5x+91=27$ Add 6x to both sides

$-5x=-64$ Subtract 91 from both sides

$x=\frac{64}{5}$ Divide both sides by -5

Practice Problems

Solve.

1. $5c-9=8-2c$

2. $2-3(x+4)=8$

3. $\frac{1}{2}\left(2x+\frac{3}{4}\right)-\frac{1}{3}\left(\frac{1}{10}+x\right)=1$

4. $\frac{8}{3}(6x+7)-8x=\frac{1}{2}(4-x)+11$

5. $10(1-2y)=-5(2y-1)$

7. $\frac{4}{7}(28n-35)=\frac{2}{3}(24n-30)$

6. $\frac{x}{2}+\frac{x}{5}+\frac{x}{3}=31$

8. $-\frac{4}{3}w+2=14-\frac{1}{3}w$

Solving a formula for one of its variables

EX1: Solve $\frac{x}{a}+1=\frac{x}{b}$ for x

$ab\left(\frac{x}{a}\right)+ab(1)=ab\left(\frac{x}{b}\right)$ Multiply each side by the LCD of ab

$bx+ab=ax$ Simplify

$ab=ax-bx$ Collect terms with an x on one side

$ab=x(a-b)$ Factor out an x

$x=\frac{ab}{a-b}$ Divide both sides by (a – b).

Practice Problems

Solve each equation for x.

1. $ax + bx - 15 = 0$

2. $\frac{x}{a} - 5 = b$

3. $\frac{x-2}{2} = m + n$

4. $\frac{x+a}{b} + b = a$

Solving Inequalities

- The properties for solving inequalities are similar to the properties for solving equations. The exception occurs when you multiply or divide each side by a negative quantity. When multiplying or dividing by a negative quantity you must switch the direction of the inequality!!

EX1: Solve $-2x < 3(x-5)$

$-2x < 3x - 15$ Distributive property
$-5x < -15$ Subtract 3x from both sides
$x > 3$ Divide both sides by -5 and switch the direction of the inequality

EX2: Solve $8a - 15 > 73$

$8a > 88$ Add 15 to both sides
$a > 11$ Divide both sides by 8. No need to switch the direction of the inequality.

EX3: Solve $2x - 3 > 2(x-5)$

$2x - 3 > 2x - 10$ Distributive property
$2x > 2x - 7$ Add 3 to both sides
$0 > -7$ Subtract 2x from both sides

The last statement is always true. Therefore the answer can be any real number. This is a special circumstance and does not happen often.

EX 4: Solve $7x + 6 < 7x - 28$

$6 < -28$ Subtract 7x from both sides

The last statement is never true. Therefore there is no solution. This is another special circumstance and does not happen often.

Practice Problems

Solve each inequality.

1. $14 - 4y \geq 38$

2. $2 - 3z \geq 7(8 - 2z) + 12$

3. $\frac{2}{3}(x - 12) \leq x + 8$

4. $-7(3x - 7) + 21x > 50$

5. $6\left[5y-(3y-1)\right]\geq 4(3y-7)$

6. The sum of the length of any 2 sides of a triangle is greater than the length of the third side. In $\triangle ABC$, BC = 4 and AC = 8 – AB. Write an inequality for AB.

Compound Inequalities

- A compound inequality is a pair of inequalities joined by *and* or *or.*
- EX: $-1<x$ and $x\leq 3$ can also be written as $-1<x\leq 3$
- EX: $x<-1$ or $x\geq 3$
- To solve a compound inequality containing *and*, find all values of the variable that make both inequalities true
- To solve a compound inequality containing *or*, find all values of the variable that make at least one of the inequalities true

EX1: Solve $3x-1>-28$ and $2x+7<19$

$3x>-27$ and $2x<12$

$x>-9$ and $x<6$

This solution can be written as $-9<x<6$

EX2: Solve $4y-2\geq 14$ or $3y-4\leq -13$

$4y\geq 16$ or $3y\leq -9$

$y\geq 4$ or $y\leq -3$

Practice Problems
Solve.

1. $2x>-10$ and $9x<18$

2. $6b+3<15$ or $4b-2>18$

3. $-6 < 2x - 4 < 12$

5. $-18 > 4x - 3 \geq -15$

4. $4x \leq 12$ or $-7x \leq 21$

6. $\frac{3}{8}x < -6$ or $5x > 2$

Absolute Value Equations

- The absolute value of a number is its distance from zero on the number line and distance is nonnegative. So the absolute value of a negative number such as -5 is its opposite. |-5| = 5.

EX1: Solve $|2y - 4| = 12$. (When solving an absolute value equation, you must set up 2 equations!!!!)

$2y - 4 = 12$	or	$2y - 4 = -12$
$2y = 16$	or	$2y = -8$
$y = 8$	or	$y = -4$

You must check your solution when solving absolute value equations.

EX2: Solve $3|4x - 1| - 5 = 10$ (To make things easier, isolate the absolute value expression before setting up 2 equations).

$3|4x - 1| = 15$ add 5 to both sides

$|4x - 1| = 5$ divide both sides by 3

Now we break this into 2 equations.

$4x - 1 = 5$	or	$4x - 1 = -5$
$4x = 6$	or	$4x = -4$
$x = \frac{3}{2}$	or	$x = -1$

Both answers check.

- The equation $|2x+7|=-2$ has no solution because $|2x+7|$ cannot be negative.
- An **extraneous solution** of an equation is derived from an original equation that is not a solution of the original equation.

EX3: Solve $|2x+5|=3x+4$

$2x+5=3x+4$ or $2x+5=-(3x+4)$

$1=x$ or $2x+5=-3x-4$

$5x=-9$

$x=-\frac{9}{5}$

Check the solution $x=-\frac{9}{5}$

$$\left|2\left(-\frac{9}{5}\right)+5\right|\stackrel{?}{=}3\left(-\frac{9}{5}\right)+4$$

$$\left|\frac{7}{5}\right|\stackrel{?}{=}-\frac{7}{5}$$

$$\frac{7}{5}\neq-\frac{7}{5}$$

Therefore $x=-\frac{9}{5}$ is an extraneous solution. The only solution to this equation is $x=1$.

Practice Problems

Solve and check.

1. $|3x|=18$

2. $|3x+4|=-18$

3. $|2y-4|=12$

4. $5|4y+6|-8=12$

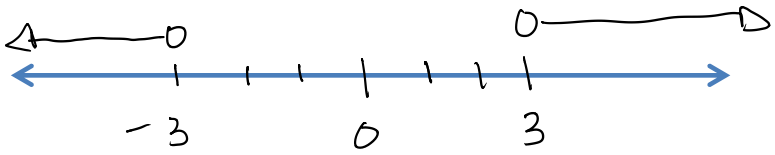
5. $|2x+5|=3x+4$

7. $6|2x+5|=6x+24$

6. $5|6-5x|=15x-35$

Absolute Value Inequalities

If $|x|>3$, then x is more than 3 units from 0 on the number line.



This is also the graph of $x<-3$ or $x>3$. So the absolute value inequality $|x|>3$ can be rewritten as the compound inequality $x<-3$ or $x>3$.

Solving inequalities of the form: $|A|\geq b$

EX1: Solve $|3x+6|\geq 12$

$3x+6\geq 12$	or	$3x+6\leq -12$	Rewrite as a compound inequality
$3x\geq 6$	or	$3x\leq -18$	
$x\geq 2$	or	$x\leq -6$	

When an absolute value is combined with other operations, first isolate the absolute value expression on one side of the inequality.

Absolute value Inequalities

Let k represent a positive real number

- $|x| \geq k$ is equivalent to $x \leq -k$ or $x \geq k$
- $|x| \leq k$ is equivalent to $-k \leq x \leq k$
- Similar to absolute value equations, an absolute value inequality must be separated into 2 inequalities.

Solving inequalities of the form $|A| < b$

EX2: Solve $3|2x+6|-9<15$

$|2x+6|<8$ Isolate the absolute value expression. Add 9 to both sides, then divide both sides by 3.
$-8<2x+6<8$ Rewrite as a compound *AND* inequality
$-14<2x<2$ Subtract 6 from all locations
$-7<x<1$ Divide all locations by 2

Practice Problems

Solve.

1. $|x+3|>9$

2. $3|y+9| \leq 27$

3. $\frac{1}{4}|y-3|+2<1$

4. $-2|x+4|<22$

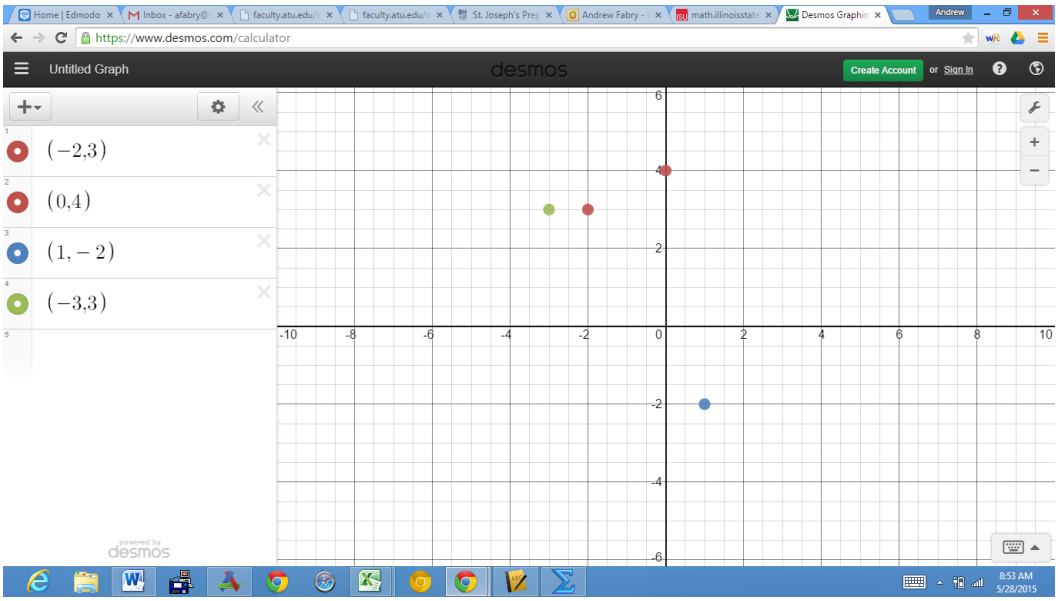
5. $|-2x+1| \geq 2$

6. $\left|\frac{x+5}{3}\right| - 3 > 6$

Relations and Functions

A **relation** is a set of pairs of input and output values (a set of (x,y) coordinate pairs).

EX1: Let Z be the relation Z = {(-2,3), (0,4), (1, -2), (-3, -3)}. We can plot the points of this relation.



The **domain** of a relation is the set of all input or *x-coordinates.*

The **range** of a relation is the set of all output or *y-coordinates.*

EX2: Find the domain and range of the relation Z

Domain of Z = {-2, 0, 1, -3}

Range of Z = {3, 4, -2, 3}

A **function** is a relation in which each element of the domain is paired with exactly one element of the range.

EX3: Determine whether each relation is a function.

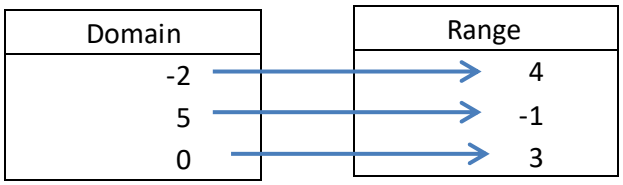
F = {(-2, 4), (5, -1), (0, 3)}

G = {(0, 5), (1, 5), (10, 5)}

H = {(-2, 4), (-2, 3), (-2, 7)}

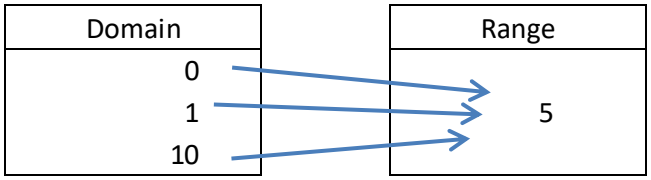
To determine if a relation is a function you can use a mapping diagram. A **mapping diagram** links the elements of the domain with the corresponding elements of the range.

Mapping diagram for F



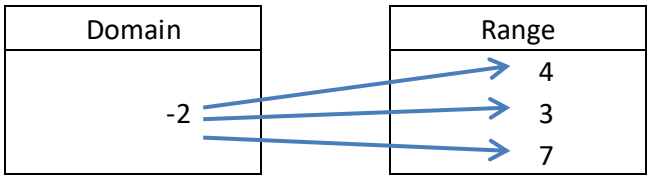
Every element of the domain is linked with one and only one element of the range. Therefore F is a function.

Mapping diagram for G



Every element of the domain is linked with one and only one element of the range. Therefore G is a function.

Mapping diagram for H



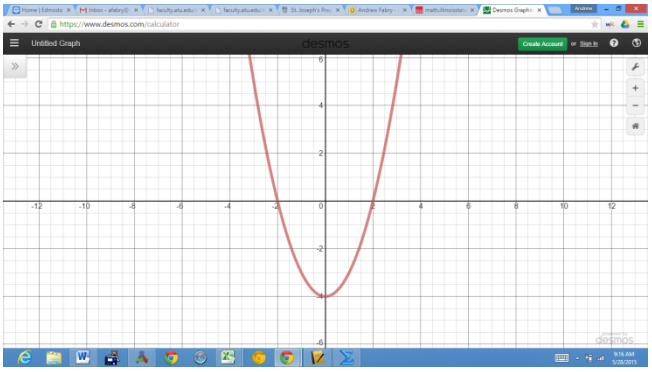
Because the domain element of -2 is linked with more than element in the range, H is not a function.

In general a relation fails to be a function if there are repeated x – values!

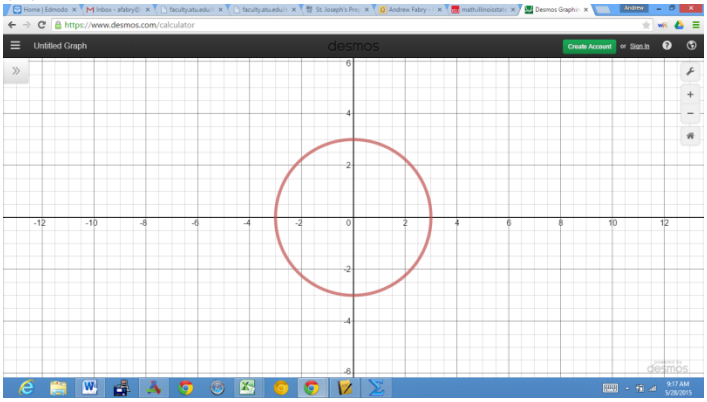
Graphing a relation on a coordinate plane gives you a visual way to tell whether it is a function. You can use the **vertical line test** to determine whether the relation has at least one element of the domain paired with more than one element of the range. If a vertical line passes through two or points on the graph, then the relation is *not* a function.

EX4: Use the vertical line test to determine which graph represents a function

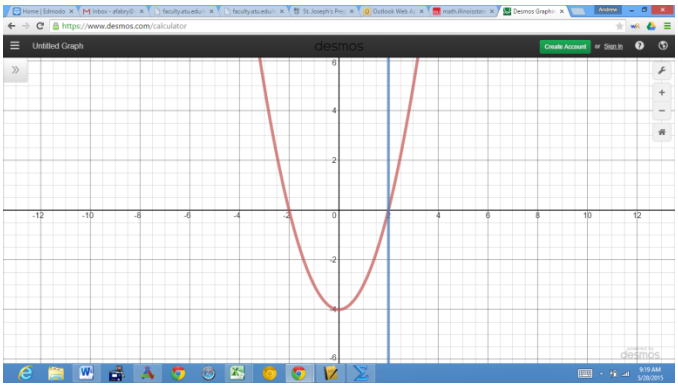
(a)



(b)

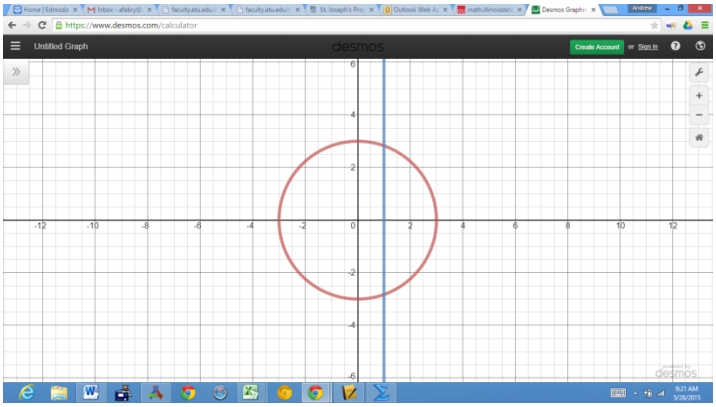


Graph (a) is the only graph that passes the vertical line test.



The vertical line only hits the graph once.

The graph from (b) fails the vertical line test.



A function rule expresses an output value in terms of an input value.

Examples of function rules:

$y = 2x$, x is the input, y is the output

$f(x) = x + 5$, x is the input, f(x) is the output

$C = \pi d$, d is the input, C is the output

You read the **function notation** $f(x)$ as "f of x." Note that $f(x)$ does not mean "f times x." When the value of x is 3, $f(3)$, read "f of 3," represents the value of the function at x = 3 ($f(x)$ is a fancy way to write y).

EX: Given $f(x) = 4x - 5$, evaluate $f(-2)$

$f(-2) = 4(-2) - 5$
$f(-2) = -8 - 5$
$f(-2) = -13$
Now we know the point $(-2,-13)$ is a point on the graph of $f(x) = 4x - 5$.

Practice Problems

1. Graph the relation

$\{(2.5,0),(-.5,0),(0,0)(2,0)\}$

2. Find the domain and range of the following relation: $\{(2.5,0),(-.5,0),(0,0)(2,0)\}$

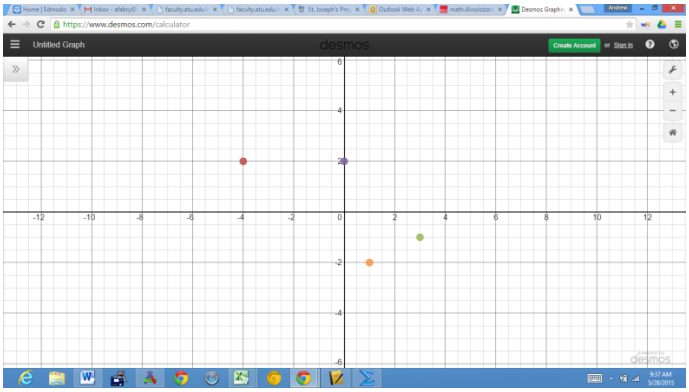
3. Determine whether each relation is a function

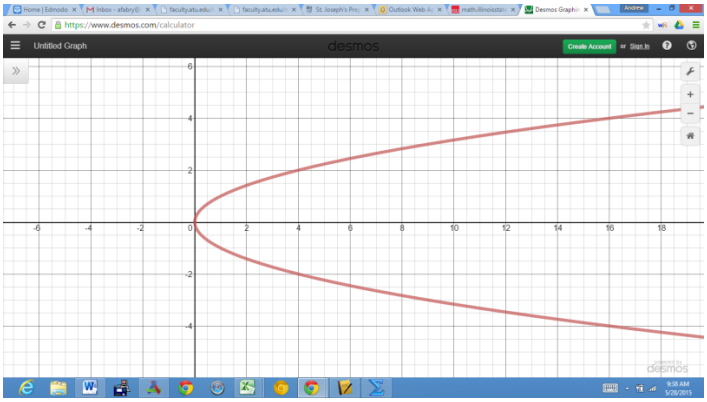
$\{(2.5,0),(-.5,0),(0,0)(2,0)\}$

$\{(-2,8),(-1,1),(-3.3,4),(7,-125)\}$

$\{(-1,41),(27,90),(-1,42.5),(9,0)\}$

4. Use the vertical line test to determine if the graph represents a function





5. For each function find $f(-5)$, $f(-3)$, $f\left(\frac{1}{2}\right)$, $f(4)$,

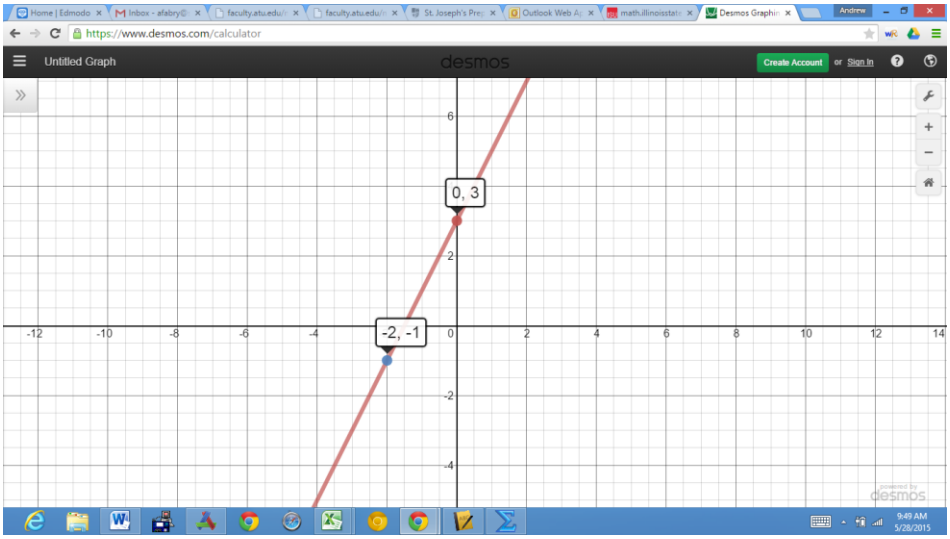
(a) $f(x)=-6x-\frac{2}{3}$ (b) $f(x)=x^2-4x+2$ (c) $f(x)=|x-5|+9$

6. Suppose $f(x)=2x+5$ and $g(x)=-\frac{1}{3}x+2$. Find each value.

(a) $\frac{f(1)}{g(3)}$ (b) $-2g(7)$ (c) $g(f(-3))$

Linear Equations

A function whose graph is a line is a **linear function**. You can represent a linear function with a **linear equation**, such as $y=2x+3$. A solution of a linear equation is any ordered pair (x, y) that makes the equation true.



The **y – intercept** of a line is the point at which the line crosses the y – axis. To find the y – intercept, set the x – coordinate equal to zero and solve for y.

The **x – intercept** of a line is the point at which the line crosses the x – axis. To find the x – intercept, set the y – coordinate equal to zero and solve for x.

EX1: Find the x and y – intercepts of the line $y = -5x + 10$.

X – intercept: set y = 0

$0 = -5x + 10$

$-10 = -5x$

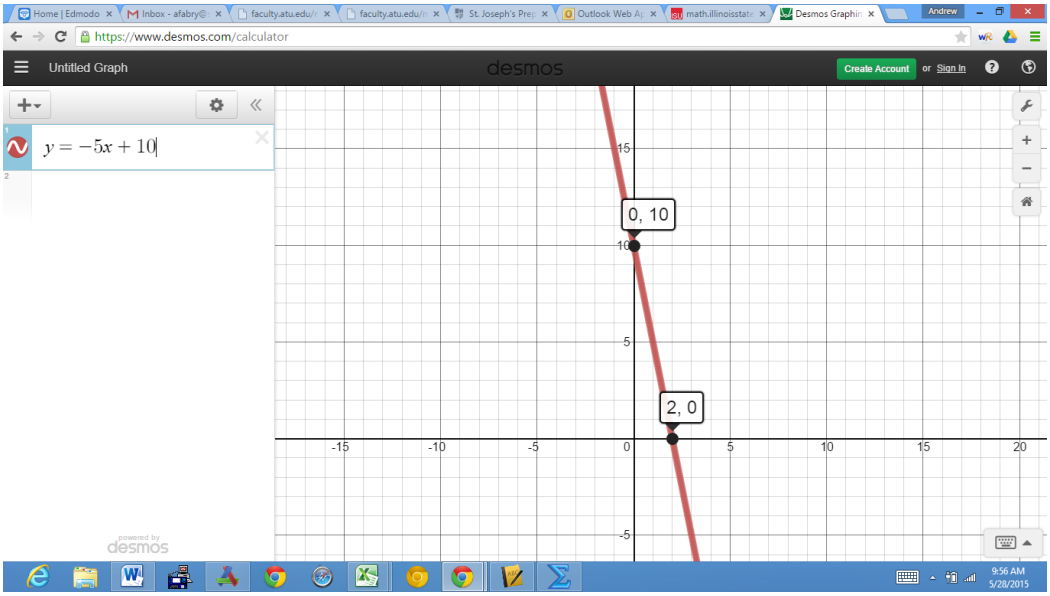
$2 = x$

Y – intercept: set x = 0

$y = -5(0) + 10$

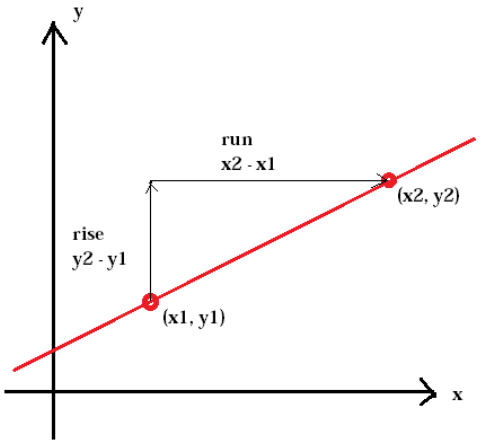
$y = 10$

Now we can graph.



The **standard form of a linear equation** is $Ax + By = C$, where A, B, and C are real numbers and A and B are not both zero. You can graph a line that is in standard form using the x and y – intercepts.

The **slope** of a non-vertical line is the ratio of the vertical change to a corresponding horizontal change.



Slope Formula

Slope is denoted m

$m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_2 - x_1 \neq 0$

EX2: Find the slope between the two points $(3,2) and (-9,6)$

$$m = \frac{6-2}{-9-3}$$

$$m = \frac{4}{-12}$$

$$m = -\frac{1}{3}$$

When you know the slope and a point on the line, you can use the **point – slope formula** to write the equation of the line.

Point – Slope Formula: The line through point (x_1, y_1) with a slope of m has the equation

$$y - y_1 = m(x - x_1)$$

EX3: Write the equation of a line with a slope of $-\frac{1}{2}$ and contains the point $(8,-1)$. Leave your answer in standard form.

Use $y-y_1=m(x-x_1)$

$y-(-1)=-\frac{1}{2}(x-8)$

$y+1=-\frac{1}{2}x+4$

$y=-\frac{1}{2}x+3$

$\frac{1}{2}x+y=3$

Another form of the equation of a line is **slope intercept form** which you can use to find the slope and the y-intercept

Slope intercept form: $y=mx+b$

m: slope

b: y - intercept

EX4: Find the slope and y-intercept of the line $-3x+4y=8$.

Translate the line from standard form into slope intercept form.

$-3x+4y=8$

$4y=3x+8$ add 3x to both sides

$y=\frac{3}{4}x+2$ divide both sides by 4

$m=\frac{3}{4}$

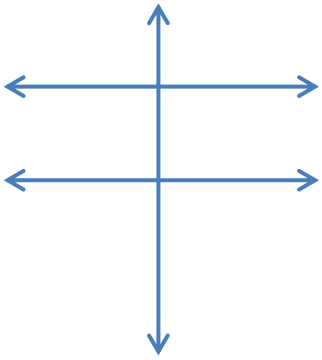
y – intercept= $(0,2)$

Summary: Equations of lines

Point Slope Form	Standard Form	Slope – Intercept Form
$y-y_1=m(x-x_1)$	$Ax+By=C$	$y=mx+b$

The slopes of horizontal, vertical, perpendicular, and parallel lines have special properties

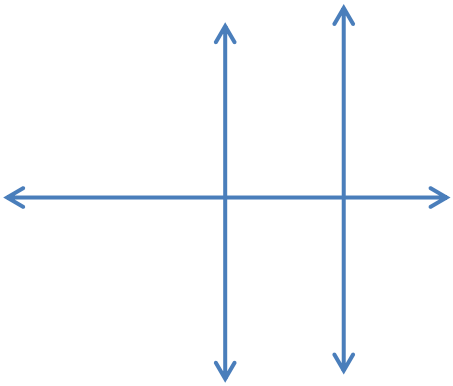
Horizontal Line



$m = 0$

y = b

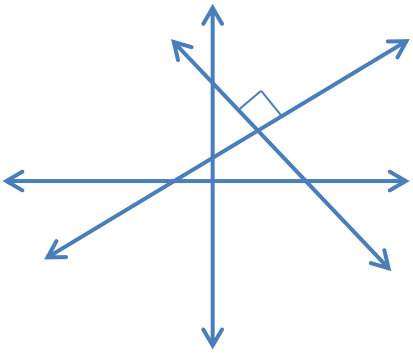
Vertical Line



m = undefined

x = x coordinate of the x - intercept

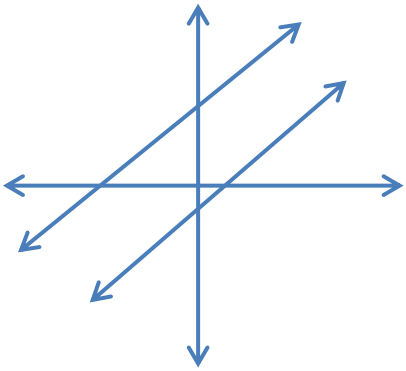
Perpendicular Lines



$m_1 \cdot m_2 = -1$

(m_2 is the negative reciprocal of m_1)

Parallel Lines



Parallel lines have the same slope but different y - intercepts

EX5: Write an equation of the line through each point and perpendicular to $y = \frac{3}{4}x + 2$ and contains the point $(0,4)$.

Leave your answer in slope intercept form.

Find the negative reciprocal of $\frac{3}{4}$

$m_{\perp} = -\frac{4}{3}$

Use the point slope formula

$y - 4 = -\frac{4}{3}(x - 0)$

$y = -\frac{4}{3}x + 4$ solve for y.

EX6: Write the equation of the line through the points $(-4,3),(5,3)$.

$$m=\frac{3-3}{5-(-4)}$$
$$m=\frac{0}{9}$$
$$m=0$$

When you get a 0 in the numerator when calculating slope means you have a horizontal line.
The slope of this line is 0 (the slope of any horizontal line is 0).
The equation for this line is $y=3$ (the equation for any horizontal line is y = the y coordinate).

EX7: Write the equation of the line through the points $(2,-8),(2,-4)$.

$$m=\frac{-4-(-8)}{2-2}$$
$$m=\frac{-4+8}{0}$$
$$m=\frac{4}{0}$$

When you get a 0 in the denominator when calculating slope means you have a vertical line.
The slope of this line is undefined (the slope of any vertical line is undefined).
The equation for this line is $x=2$ (the equation for any vertical line is $x=$ the x coordinate).

Practice Problems

Graph each equation. Label the x and y – intercepts.

1. $y=2x$

2. $y=-3x-1$

3. $5x-2y=-4$

4. $y-3=-\frac{4}{3}x$

5. $y=9$

6. $x=-3$

Find the slope of the line through each pair of points.

7. $(1,6),(8,-1)$

8. $(-5,-7),(0,10)$

9. $\left(\frac{2}{3},\frac{4}{7}\right),\left(\frac{2}{3},\frac{11}{7}\right)$

10. $(-3,5),(4,5)$

Find the equation of the line. Leave your answer in standard form.

11. $m=3;(1,5)$

12. $m=-\frac{3}{5};(-4,0)$

Find the equation of the line that contains the given points. Leave your answer in slope intercept form.

13. $(-10,3),(-2,-5)$

14. $(1,9),(6,2)$

15. $(7,3),(-2,3)$

Find the slope of each line

16. $Ax+By=C$

17. $-\frac{1}{2}x-y=-\frac{3}{4}$

Write an equation of the line. Leave your answer in slope intercept form

18. Through $(-2,1)$ and parallel to $y=-3x+1$

19. Through $(-3,-1)$ and perpendicular to $y=-\frac{2}{5}x-4$

20. Through $(0,4)$ and perpendicular to $y=4$

21. Through $(-8,4)$ and parallel to $x=-3$

22. Put into standard form: $y=\frac{3}{2}x-1$

23. Prove that a triangle with vertices $(3,5),(-2,6),(1,3)$ is a right triangle.

Direct Variation

A liner function defined by an equation of the form $y = kx$, where $k \neq 0$, represents **direct variation**. As with any line, the slope, k is constant.

Solving for k we get, $k = \frac{y}{x}$. k is called the **constant of variation.**

EX1: For each function, determine whether y varies directly with x. If so, find the constant of variation and write the equation.

(a)

x	y
2	8
3	12
5	20

$\frac{y}{x} = \frac{8}{2} = \frac{12}{3} = \frac{20}{5} = 4$

So y varies directly with x.
The constant of variation, k = 4.
The equation is y = 4x.

(b)

x	Y
1	4
2	7
5	16

Since $\frac{4}{1} \neq \frac{7}{2} \neq \frac{16}{5}$

y does not vary directly with x.
This is not a direct variation.

EX2: For each function, determine whether y varies directly with x If so, find the constant of variation.

(a) $3y = 2x$

Solve for y.

$y = \frac{3}{2}x$

This is of the form $y = kx$.

The constant of variation is $\frac{3}{2}$.

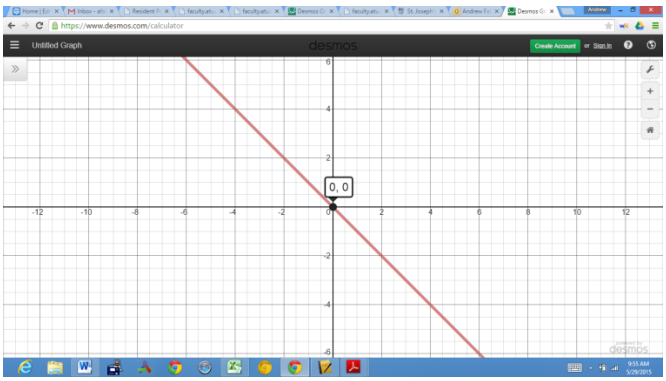
(b) $y = 2x + 3$

This is not of the form $y = kx$ (the +3 violates it).

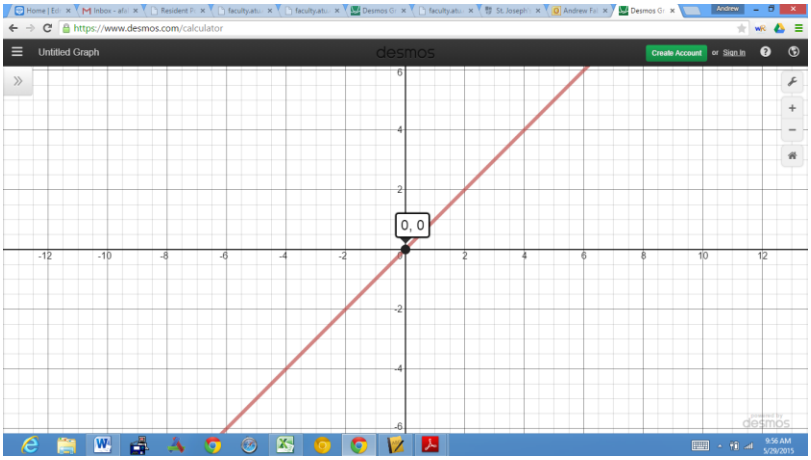
This is not a direct variation.

The graph of a direct variation must pass through the origin.

For k < 0



For k>0



Practice Problems

For each function, determine whether y varies directly with x. If so find the constant of variation and write the equation.

1.

x	y
3	9
4	13
7	23

2.

x	y
2	4
4	8
16	32

For each of the following, y varies directly with x.

3. If y = 4 when x = -2, find x when y = 6.

4. If y = 6 when x = 2, find x when y = 12.

5. If y = .9 when x = 4.8, find y when x = 6.4.

Determine whether a line with the given slope through the given point represents a direct variation.

6. $m=-1.7,(9,-9)$

7. $m=-\frac{5}{6},\left(15,-\frac{25}{2}\right)$

Answers to practice problems.

Solving equations

1. $c=\frac{17}{7}$ 2. $x=-6$ 3. $x=\frac{79}{80}$ 4. $x=-\frac{2}{3}$

5. $y=\frac{1}{2}$ 6. $x=30$ 7. All real numbers 8. $w=-12$

Solving a formula for one of its variables

1. $x=\frac{15}{(a+b)}$ 2. $x=ab+5a$ 3. $x=2(m+n)+2$

4. $x=ab-b^2-a$

Solving Inequalities

1. $y\le -6$ 2. $z\ge 6$ 3. $x\ge -48$ 4. No solution

5. All real numbers 6. $2<AB<6$

Compound Inequalities

1. $-5<x<2$ 2. $b<2 \text{ or } b>5$ 3. $-1<x<8$

4. All real numbers 5. No solution 6. $x<-16 \text{ or } x>\frac{2}{5}$

Absolute Value Equations

1. $x=6, x=-6$ 2. No solution 3. $y=8, y=-4$

4. $y=-\frac{1}{2}, y=-\frac{5}{2}$ 5. $x=1$ 6. No solution

7. $x=-1, x=-3$

Absolute Value Inequalities

1. $x<-12 \text{ or } x>6$ 2. $-18\le y\le 0$ 3. No solution

4. All real numbers 5. $x\le -\frac{1}{2} \text{ or } x\ge \frac{3}{2}$ 6. $x<-32 \text{ or } x>22$

Relations and Functions

2. D: {2.5, -.5, 0, 2} R: {0} 3. Function, function, not a function

4. function, not a function

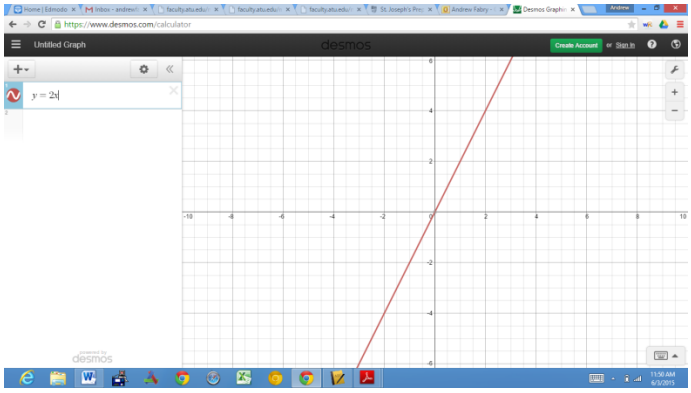
5. (a) $f(-5)=29\frac{1}{3},\ f(-3)=17\frac{1}{3},\ f\left(\frac{1}{2}\right)=-3\frac{2}{3},\ f(4)=-24\frac{2}{3}$

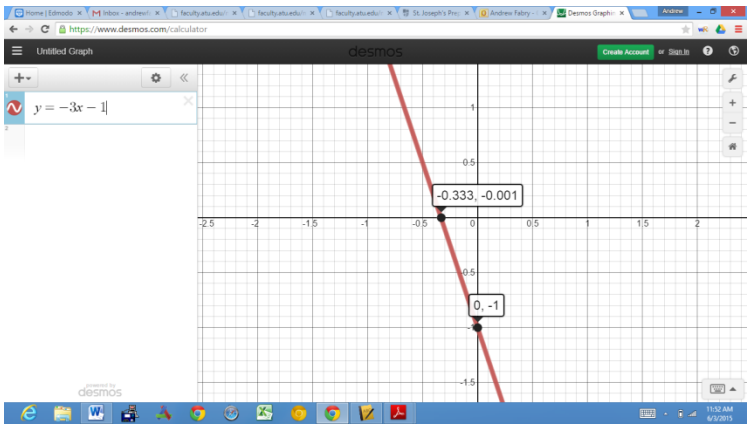
(b) $f(-5)=47,\ f(-3)=23,\ f\left(\frac{1}{2}\right)=\frac{1}{4},\ f(4)=2$

(c) $f(-5)=19,\ f(-3)=17,\ f\left(\frac{1}{2}\right)=13.5,\ f(4)=10$

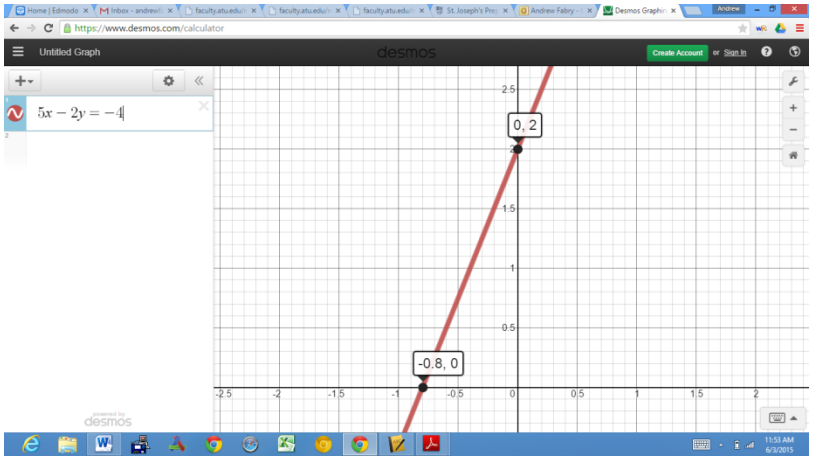
6. (a) 7 (b) $\frac{2}{3}$ (c) $\frac{7}{3}$

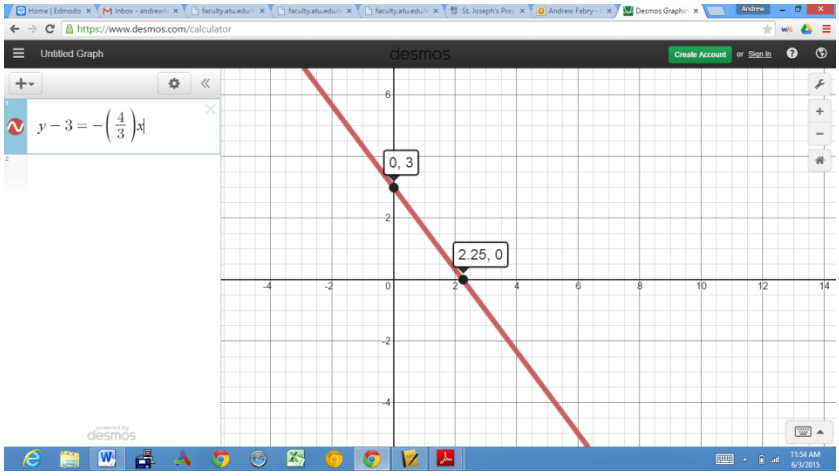
Linear Equations

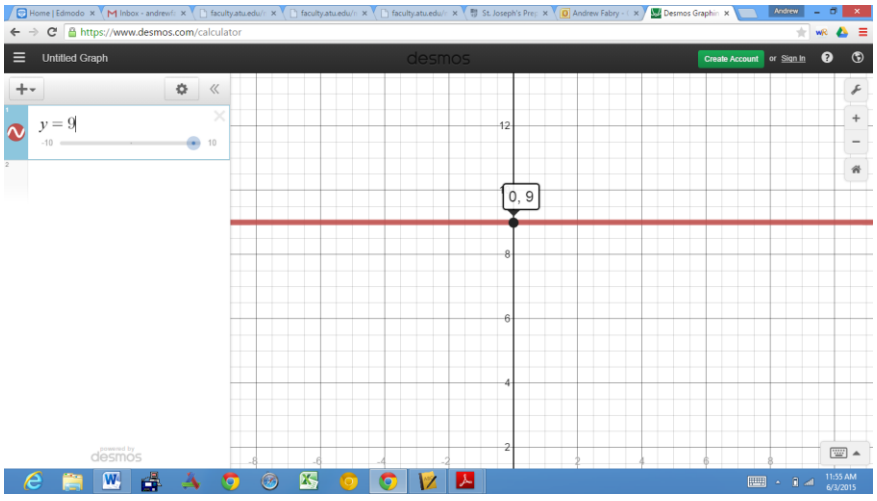
1. 

2. 

the x – intercept here is $\left(-\frac{1}{3},0\right)$.

3. 
Untitled Graph
desmos
Create Account
Sign In
$5x - 2y = -4$
0, 2
-0.8, 0

4. 
Untitled Graph
desmos
Create Account
Sign In
$y - 3 = -\left(\frac{4}{3}\right)x$
0, 3
2.25, 0

5. 
Untitled Graph
desmos
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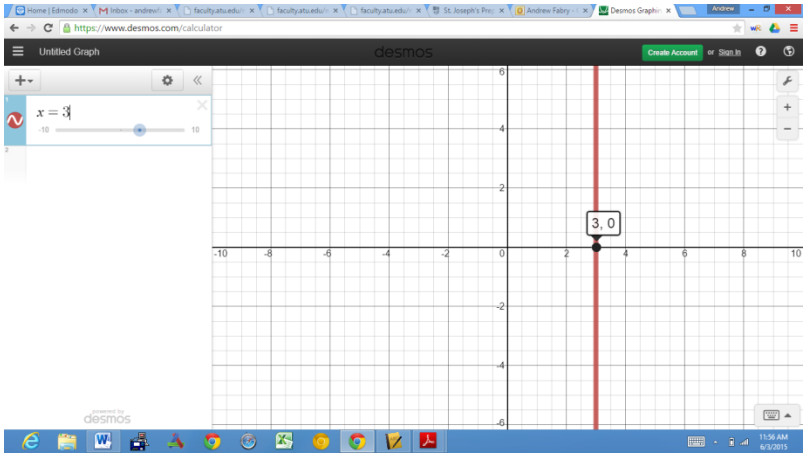
Sign In
$y = 9$
0, 9

6. 

7. $m=-1$ 8. $m=\frac{17}{5}$ 9. Undefined 10. $m=0$

11. $-3x+y=2$ 12. $3x+5y=-12$ 13. $y=-x-7$

14. $y=-\frac{7}{5}x+\frac{52}{5}$ 15. $y=3$ 16. $m=-\frac{A}{B}$ 17. $m=-\frac{1}{2}$

18. $y=-3x-5$ 19. $y=\frac{5}{2}x+\frac{13}{2}$ 20. X = 0

21. x = -8

22. $-3x+2y=-2$

23. The slope between points (-2, 6) and (1, 3) is -1
The slope between points (1, 3) and (3, 5) is 1. 1 and -1 are negative reciprocal. There the triangle is a right triangle.

Direct Variation

1. Not a direct variation 2. This is a direct variation: y = 2x

3. x = -3 4. x = 4 5. y = 1.2

6. Not a direct variation 7. $y=-\frac{5}{6}x$. This is a direct variation