St. Joseph's Prep

Department of Mathematics

Algebra II/Trigonometry

Summer Review Packet

Instructions: Complete this review packet in its entirety. All of this content should be review. Be prepared to ask questions on this review packet the first day of classes. There will be a test on the first B day of the first quarter. You may use a calculator.

A. Simplifying Polynomial Expressions

I. Combining Like Terms

- You can add or subtract terms that are considered "like", or terms that have the same variable(s) with the same exponent(s).

Ex. 1: $5x - 7y + 10x + 3y$

$\underline{5x} \underline{- 7y} + \underline{10x} + \underline{3y}$

$15x - 4y$

Ex. 2: $-8h^2 + 10h^3 - 12h^2 - 15h^3$

$\underline{-8h^2} + \underline{10h^3} \underline{- 12h^2} \underline{- 15h^3}$

$-20h^2 - 5h^3$

II. Applying the Distributive Property

- Every term inside the parentheses is multiplied by the term outside of the parentheses.

Ex. 1: $3(9x-4)$

$3 \cdot 9x - 3 \cdot 4$

$27x - 12$

Ex. 2: $4x^2(5x^3 + 6x)$

$4x^2 \cdot 5x^3 + 4x^2 \cdot 6x$

$20x^5 + 24x^3$

III. Combining Like Terms AND the Distributive Property (Problems with a Mix!)

- Sometimes problems will require you to distribute AND combine like terms!!

Ex. 1: $3(4x-2)+13x$

$3 \cdot 4x - 3 \cdot 2 + 13x$

$12x - 6 + 13x$

$25x - 6$

Ex. 2: $3(12x-5) - 9(-7+10x)$

$3 \cdot 12x - 3 \cdot 5 - 9(-7) - 9(10x)$

$36x - 15 + 63 - 90x$

$-54x + 48$

PRACTICE SET 1

Simplify.

1. $8x - 9y + 16x + 12y$

2. $14y + 22 - 15y^2 + 23y$

3. $5n - (3 - 4n)$

4. $-2(11b - 3)$

5. $10q(16x + 11)$

6. $-(5x - 6)$

7. $3(18z - 4w) + 2(10z - 6w)$

8. $(8c + 3) + 12(4c - 10)$

9. $9(6x - 2) - 3(9x^2 - 3)$

10. $-(y - x) + 6(5x + 7)$

B. Solving Equations

I. Solving Two-Step Equations

A couple of hints:

1. To solve an equation, UNDO the order of operations and work in the reverse order.
2. REMEMBER! Addition is "undone" by subtraction, and vice versa. Multiplication is "undone" by division, and vice versa.

Ex. 1: $4x - 2 = 30$

$+2 \quad +2$

$4x = 32$

$\div 4 \quad \div 4$

$x = 8$

Ex. 2: $87 = -11x + 21$

$-21 \qquad -21$

$66 = -11x$

$\div -11 \quad \div -11$

$-6 = x$

II. Solving Multi-step Equations With Variables on Both Sides of the Equal Sign

- When solving equations with variables on both sides of the equal sign, be sure to get all terms with variables on one side and all the terms without variables on the other side.

Ex. 3: $8x + 4 = 4x + 28$

$-4 \qquad -4$

$8x = 4x + 24$

$-4x \quad -4x$

$4x = 24$

$\div 4 \quad \div 4$

$x = 6$

III. Solving Equations that need to be simplified first

- In some equations, you will need to combine like terms and/or use the distributive property to simplify each side of the equation, and then begin to solve it.

Ex. 4: $5(4x - 7) = 8x + 45 + 2x$

$20x - 35 = 10x + 45$

$-10x \qquad -10x$

$10x - 35 = 45$

$+35 \quad +35$

$10x = 80$

$\div 10 \quad \div 10$

$x = 8$

PRACTICE SET 2

Solve each equation. You must show all work.

1. $5x - 2 = 33$

2. $140 = 4x + 36$

3. $8(3x - 4) = 196$

4. $45x - 720 + 15x = 60$

5. $132 = 4(12x - 9)$

6. $198 = 154 + 7x - 68$

7. $-131 = -5(3x - 8) + 6x$

8. $-7x - 10 = 18 + 3x$

9. $12x + 8 - 15 = -2(3x - 82)$

10. $-(12x - 6) = 12x + 6$

IV. Solving Literal Equations

- A literal equation is an equation that contains more than one variable.
- You can solve a literal equation for one of the variables by getting that variable by itself (isolating the specified variable).

Ex. 1: $3xy = 18$, Solve for x.

$$\frac{3xy}{3y} = \frac{18}{3y}$$

$$x = \frac{6}{y}$$

Ex. 2: $5a - 10b = 20$, Solve for a.

$$+10b = +10b$$

$$5a = 20 + 10b$$

$$\frac{5a}{5} = \frac{20}{5} + \frac{10b}{5}$$

$$a = 4 + 2b$$

PRACTICE SET 3

Solve each equation for the specified variable.

1. $Y + V = W$, for V

2. $9wr = 81$, for w

3. $2d - 3f = 9$, for f

4. $dx + t = 10$, for x

5. $P = (g - 9)180$, for g

6. $4x + y - 5h = 10y + u$, for x

C. Rules of Exponents

Multiplication: Recall $(x^m)(x^n)=x^{(m+n)}$ *Ex:* $(3x^4y^2)(4xy^5)=(3\cdot 4)(x^4\cdot x^1)(y^2\cdot y^5)=12x^5y^7$

Division: Recall $\frac{x^m}{x^n}=x^{(m-n)}$ *Ex:* $\frac{42m^5j^2}{-3m^3j}=\left(\frac{42}{-3}\right)\left(\frac{m^5}{m^3}\right)\left(\frac{j^2}{j^1}\right)=-14m^2j$

Powers: Recall $(x^m)^n = x^{(m\cdot n)}$ *Ex:* $(-2a^3bc^4)^3 = (-2)^3(a^3)^3(b^1)^3(c^4)^3 = -8a^9b^3c^{12}$

Power of Zero: Recall $x^0 = 1,\ x \neq 0$ *Ex:* $5x^0y^4 = (5)(1)(y^4) = 5y^4$

PRACTICE SET 4

Simplify each expression.

1. $(c^5)(c)(c^2)$
2. $\frac{m^{15}}{m^3}$
3. $(k^4)^5$
4. d^0
5. $(p^4q^2)(p^7q^5)$
6. $\frac{45y^3z^{10}}{5y^3z}$
7. $(-t^7)^3$
8. $3f^3g^0$
9. $(4h^5k^3)(15k^2h^3)$
10. $\frac{12a^4b^6}{36ab^2c}$
11. $(3m^2n)^4$
12. $(12x^2y)^0$
13. $(-5a^2b)(2ab^2c)(-3b)$
14. $4x(2x^2y)^0$
15. $(3x^4y)(2y^2)^3$

D. Binomial Multiplication

I. Reviewing the Distributive Property

The distributive property is used when you want to multiply a single term by an expression.

Ex 1: $8(5x^2 - 9x)$

$8 \cdot 5x^2 + 8 \cdot (-9x)$

$40x^2 - 72x$

II. Multiplying Binomials – the FOIL method

When multiplying two binomials (an expression with <u>two</u> terms), we use the "FOIL" method. The "FOIL" method uses the distributive property <u>twice</u>!

FOIL is the order in which you will multiply your terms.

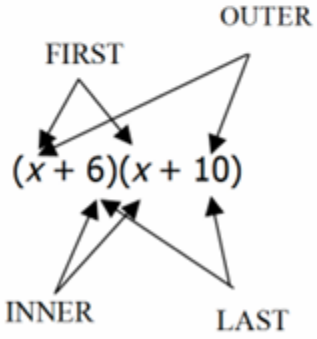
First

Outer

Inner

Last

Ex. 1: $(x + 6)(x + 10)$



First $x \cdot x$ ------> x^2

Outer $x \cdot 10$ -----> $10x$

Inner $6 \cdot x$ ------> $6x$

Last $6 \cdot 10$ -----> 60

$x^2 + 10x + 6x + 60$

$x^2 + 16x + 60$

(After combining like terms)

PRACTICE SET 5

Multiply. Write your answer in simplest form.

1. $(x + 10)(x - 9)$

2. $(x + 7)(x - 12)$

3. $(x - 10)(x - 2)$

4. $(x - 8)(x + 81)$

5. $(2x - 1)(4x + 3)$

6. $(-2x + 10)(-9x + 5)$

7. $(-3x - 4)(2x + 4)$

8. $(x + 10)^2$

9. $(-x + 5)^2$

10. $(2x - 3)^2$

E. Factoring

I. Using the Greatest Common Factor (GCF) to Factor.

- Always determine whether there is a greatest common factor (GCF) first.

Ex. 1 $3x^4 - 33x^3 + 90x^2$

- In this example the GCF is $3x^2$.
- So when we factor, we have $3x^2(x^2 - 11x + 30)$.
- Now we need to look at the polynomial remaining in the parentheses. Can this trinomial be factored into two binomials? In order to determine this make a list of all of the factors of 30.

30		30	
1	30	-1	-30
2	15	-2	-15
3	10	-3	-10
5	6	-5	-6

Since -5 + -6 = -11 and (-5)(-6) = 30 we should choose -5 and -6 in order to factor the expression.

- The expression factors into $3x^2(x-5)(x-6)$

Note: Not all expressions will have a GCF. If a trinomial expression does not have a GCF, proceed by trying to factor the trinomial into two binomials.

II. Applying the difference of squares: $a^2 - b^2 = (a-b)(a+b)$

Ex. 2 $4x^3 - 100x$

$4x(x^2 - 25)$

$4x(x-5)(x+5)$

Since x^2 and 25 are perfect squares separated by a subtraction sign, you can apply the difference of two squares formula.

PRACTICE SET 6

Factor each expression.

1. $3x^2 + 6x$

2. $4a^2b^2 - 16ab^3 + 8ab^2c$

3. $x^2 - 25$

4. $n^2 + 8n + 15$

5. $g^2 - 9g + 20$

6. $d^2 + 3d - 28$

7. $z^2 - 7z - 30$

8. $m^2 + 18m + 81$

9. $4y^3 - 36y$

10. $5k^2 + 30k - 135$

F. Graphing Lines

I. Finding the Slope of the Line that Contains each Pair of Points.

Given two points with coordinates (x_1, y_1) and (x_2, y_2), the formula for the slope, m, of the line containing the points is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Ex. (2, 5) and (4, 1)

$$m = \frac{1-5}{4-2} = \frac{-4}{2} = -2$$

The slope is -2.

Ex. (-3, 2) and (2, 3)

$$m = \frac{3-2}{2-(-3)} = \frac{1}{5}$$

The slope is $\frac{1}{5}$

PRACTICE SET 7

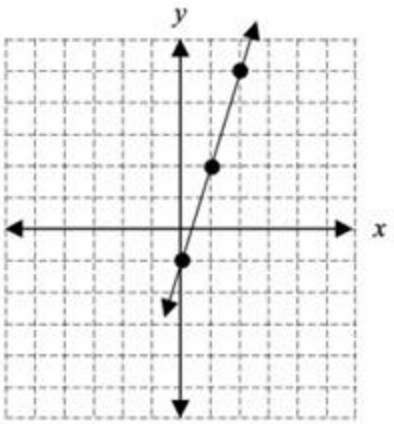
1. (-1, 4) and (1, -2)
2. (3, 5) and (-3, 1)
3. (1, -3) and (-1, -2)
4. (2, -4) and (6, -4)
5. (2, 1) and (-2, -3)
6. (5, -2) and (5, 7)

II. Using the Slope – Intercept Form of the Equation of a Line.

The slope-intercept form for the equation of a line with slope m and y-intercept b is $y = mx + b$.

Ex. $y = 3x - 1$

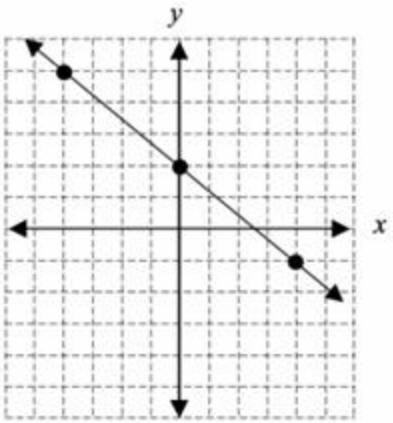
Slope: 3 y-intercept: -1



Place a point on the y-axis at -1. Slope is 3 or 3/1, so travel up 3 on the y-axis and over 1 to the right.

Ex. $y = -\frac{3}{4}x + 2$

Slope: $-\frac{3}{4}$ y-intercept: 2

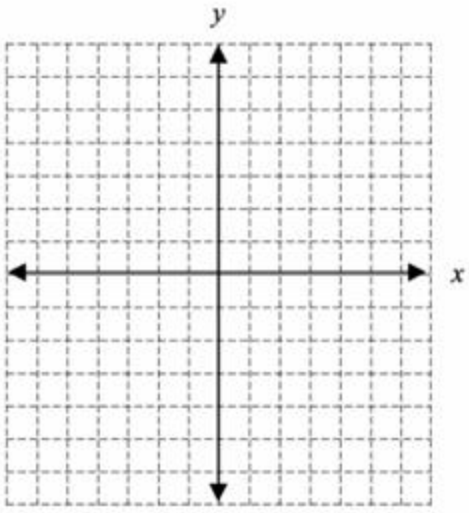


Place a point on the y-axis at 2. Slope is -3/4 so travel down 3 on the y-axis and over 4 to the right. Or travel up 3 on the y-axis and over 4 to the left.

PRACTICE SET 8

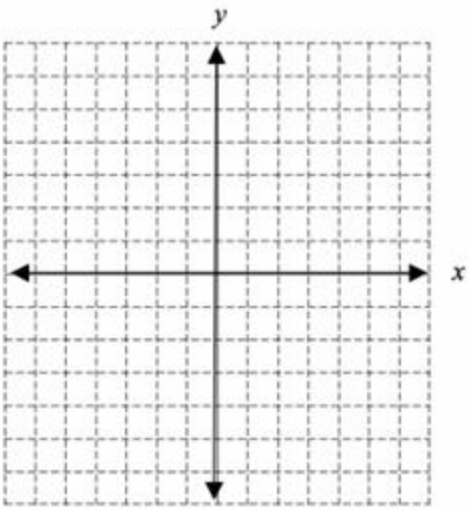
1. $y = 2x + 5$

Slope: _____ y-intercept: _____



2. $y = \frac{1}{2}x - 3$

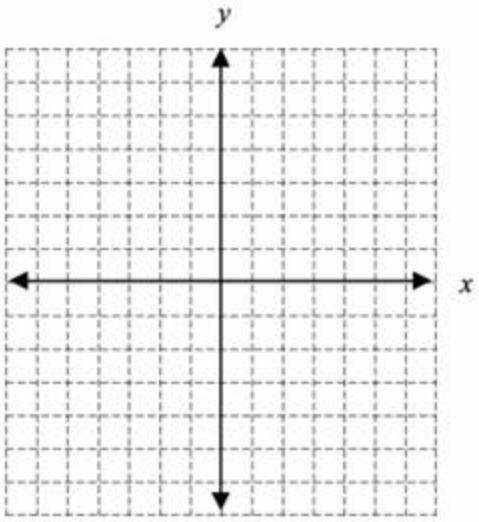
Slope: _____ y-intercept: _____



3. $y = -\frac{2}{5}x + 4$

Slope: _____

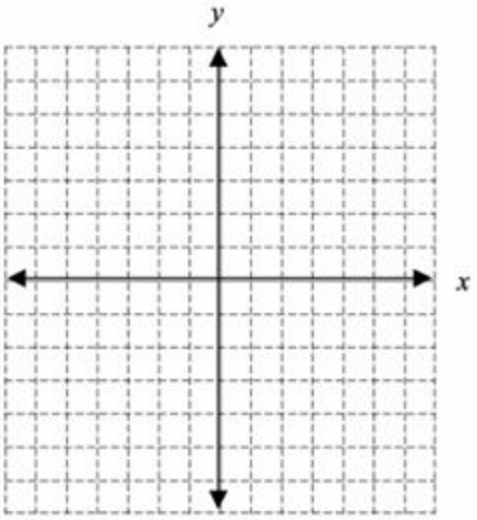
y-intercept: _____



4. $y = -3x$

Slope: _____

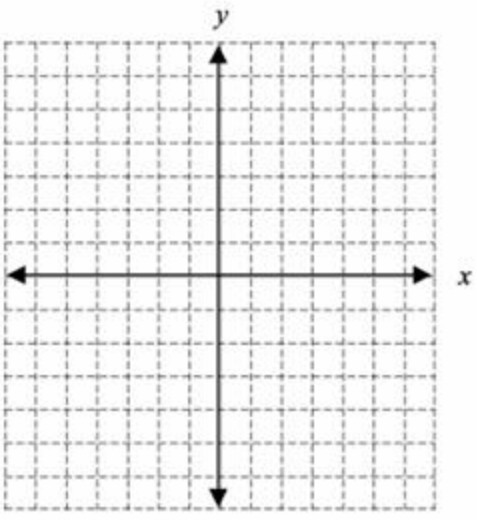
y-intercept _____



5. $y = -x + 2$

Slope: _____

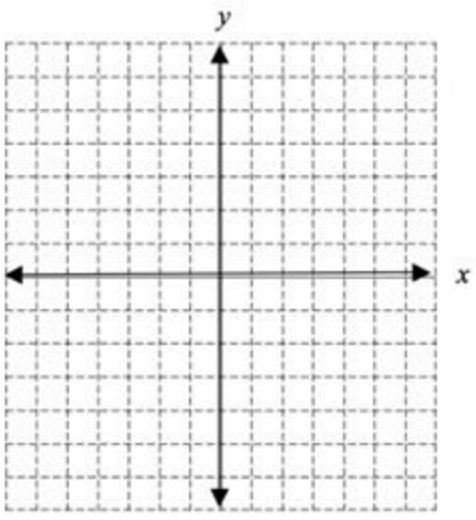
y-intercept: _____



6. $y = x$

Slope: _____

y-intercept _____



III. Using Standard Form to Graph a Line.

An equation in standard form can be graphed using several different methods. Two methods are explained below.

a. Re-write the equation in $y = mx + b$ form, identify the y-intercept and slope, then graph as in Part II above.

b. Solve for the x- and y- intercepts. To find the x-intercept, let $y = 0$ and solve for x. To find the y-intercept, let $x = 0$ and solve for y. Then plot these points on the appropriate axes and connect them with a line.

Ex. $2x - 3y = 10$

a. Solve for y.

$-3y = -2x + 10$

$y = \dfrac{-2x + 10}{-3}$

$y = \dfrac{2}{3}x - \dfrac{10}{3}$

OR

b. Find the intercepts:

let $y = 0$:

$2x - 3(0) = 10$

$2x = 10$

$x = 5$

So x-intercept is (5, 0)

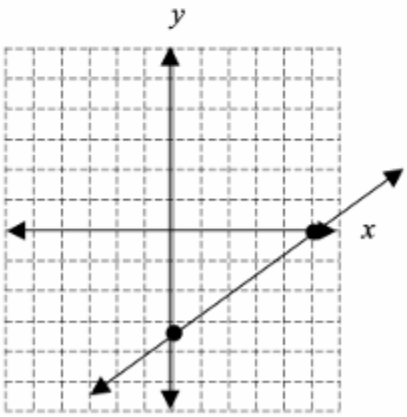
let $x = 0$:

$2(0) - 3y = 10$

$-3y = 10$

$y = -\dfrac{10}{3}$

So y-intercept is $\left(0, -\dfrac{10}{3}\right)$



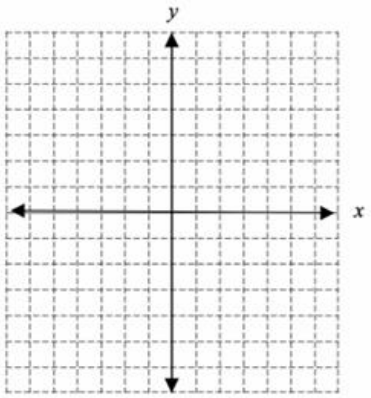
On the x-axis place a point at 5.

On the y-axis place a point at $-\dfrac{10}{3} = -3\dfrac{1}{3}$

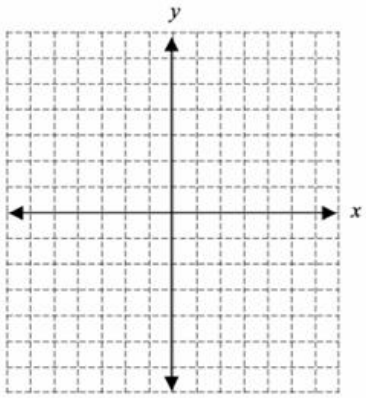
Connect the points with the line.

PRACTICE SET 9

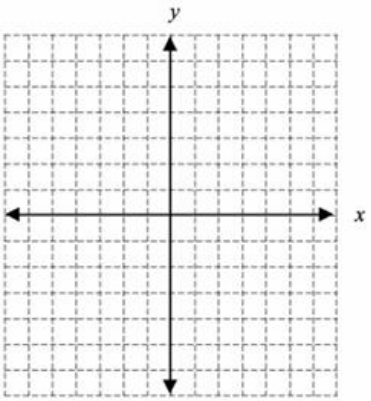
1. $3x + y = 3$



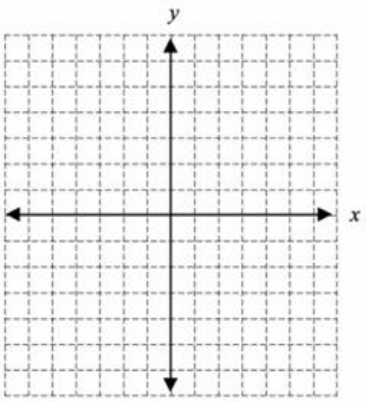
2. $5x + 2y = 10$



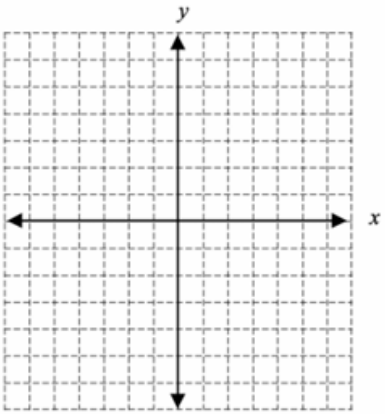
3. $y = 4$



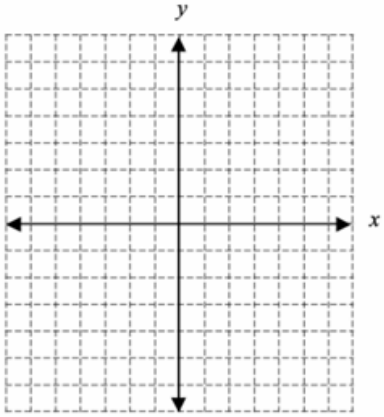
4. $4x - 3y = 9$



5. $-2x + 6y = 12$



6. $x = -3$



G. INEQUALITIES

I. Solving Inequalities

- The properties for solving inequalities are similar to the properties for solving equations. The exception occurs when you multiply or divide each side by a negative quantity. When multiplying or dividing by a negative quantity you must switch the direction of the inequality!!

EX1: Solve $-2x < 3(x-5)$

$-2x < 3x - 15$ Distributive property

$-5x < -15$ Subtract 3x from both sides

$x > 3$ Divide both sides by -5 and switch the direction of the inequality

EX2: Solve $8a - 15 > 73$

$8a > 88$ Add 15 to both sides

$a > 11$ Divide both sides by 8. No need to switch the direction of the inequality.

EX3: Solve $2x - 3 > 2(x-5)$

$2x - 3 > 2x - 10$ Distributive property

$2x > 2x - 7$ Add 3 to both sides

$0 > -7$ Subtract 2x from both sides

The last statement is always true. Therefore the answer can be any real number. This is a special circumstance and does not happen often.

EX 4: Solve $7x + 6 < 7x - 28$

$6 < -28$ Subtract 7x from both sides

The last statement is never true. Therefore there is no solution. This is another special circumstance and does not happen often.

PRACTICE SET 10

Solve each inequality.

1. $14-4y\geq 38$

2. $2-3z\geq 7(8-2z)+12$

3. $\frac{2}{3}(x-12)\leq x+8$

4. $-7(3x-7)+21x>50$

5. $6\left[5y-(3y-1)\right]\geq 4(3y-7)$

6. The sum of the length of any 2 sides of a triangle is greater than the length of the third side. In $\triangle ABC$, BC = 4 and AC = 8 – AB. Write an inequality for AB.

II. Compound Inequalities

- A compound inequality is a pair of inequalities joined by *and* or *or*.
- EX: $-1 < x$ and $x \leq 3$ can also be written as $-1 < x \leq 3$
- EX: $x < -1$ or $x \geq 3$
- To solve a compound inequality containing *and*, find all values of the variable that make both inequalities true
- To solve a compound inequality containing *or*, find all values of the variable that make at least one of the inequalities true

EX1: Solve $3x - 1 > -28$ and $2x + 7 < 19$

$3x > -27$ and $2x < 12$

$x > -9$ and $x < 6$

This solution can be written as $-9 < x < 6$

EX2: Solve $4y - 2 \geq 14$ or $3y - 4 \leq -13$

$4y \geq 16$ or $3y \leq -9$

$y \geq 4$ or $y \leq -3$

PRACTICE SET 11

Solve.

1. $2x > -10$ and $9x < 18$

2. $6b + 3 < 15$ or $4b - 2 > 18$

3. $-6 < 2x - 4 < 12$

5. $-18 > 4x - 3 \geq -15$

4. $4x \leq 12$ or $-7x \leq 21$

6. $\frac{3}{8}x < -6$ or $5x > 2$

H. ABSOLUTE VALUE

I Absolute Value Equations

- The absolute value of a number is its distance from zero on the number line and distance is nonnegative. So the absolute value of a negative number such as -5 is its opposite. |-5| = 5.

EX1: Solve $|2y-4|=12$. (When solving an absolute value equation, you must set up 2 equations!!!!)

$2y-4=12$ or $2y-4=-12$

$2y=16$ or $2y=-8$

$y=8$ or $y=-4$

You must check your solution when solving absolute value equations.

EX2: Solve $3|4x-1|-5=10$ (To make things easier, isolate the absolute value expression before setting up 2 equations).

$3|4x-1|=15$ add 5 to both sides

$|4x-1|=5$ divide both sides by 3

Now we break this into 2 equations.

$4x-1=5$ or $4x-1=-5$

$4x=6$ or $4x=-4$

$x=\frac{3}{2}$ or $x=-1$

Both answers check.

- The equation $|2x+7|=-2$ has no solution because $|2x+7|$ cannot be negative.

- An **extraneous solution** of an equation is derived from an original equation that is not a solution of the original equation.

EX3: Solve $|2x+5|=3x+4$

$2x+5=3x+4$ or $2x+5=-(3x+4)$

$1=x$ or $2x+5=-3x-4$

$5x=-9$

$x=-\frac{9}{5}$

Check the solution $x=-\frac{9}{5}$

$\left|2\left(-\frac{9}{5}\right)+5\right|\stackrel{?}{=}3\left(-\frac{9}{5}\right)+4$

$\left|\frac{7}{5}\right|\stackrel{?}{=}-\frac{7}{5}$

$\frac{7}{5}\neq-\frac{7}{5}$

Therefore $x=-\frac{9}{5}$ is an extraneous solution. The only solution to this equation is $x=1$.

PRACTICE SET 12

Solve and check.

1. $|3x|=18$

2. $|3x+4|=-18$

3. $|2y-4|=12$

4. $5|4y+6|-8=12$

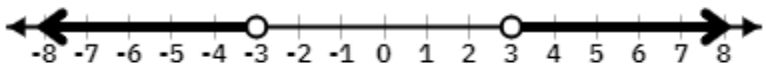
5. $|2x+5|=3x+4$

6. $5|6-5x|=15x-35$

7. $6|2x+5|=6x+24$

II. Absolute Value Inequalities

If $|x|>3,$ then x is more than 3 units from 0 on the number line.



This is also the graph of $x<-3$ or $x>3$. So the absolute value inequality $|x|>3$ can be rewritten as the compound inequality $x<-3$ or $x>3$.

Solving inequalities of the form: $|A|\geq b$

EX1: Solve $|3x+6|\geq 12$

$3x+6\geq 12$	or	$3x+6\leq -12$	Rewrite as a compound inequality
$3x\geq 6$	or	$3x\leq -18$	
$x\geq 2$	or	$x\leq -6$	

When an absolute value is combined with other operations, first isolate the absolute value expression on one side of the inequality.

Absolute value Inequalities

Let k represent a positive real number

- $|x|\geq k$ is equivalent to $x\leq -k$ or $x\geq k$
- $|x|\leq k$ is equivalent to $-k\leq x\leq k$
- Similar to absolute value equations, an absolute value inequality must be separated into 2 inequalities.

Solving inequalities of the form $|A|<b$

EX2: Solve $3|2x+6|-9<15$

$|2x+6|<8$ Isolate the absolute value expression. Add 9 to both sides, then divide both sides by 3.

$-8<2x+6<8$ Rewrite as a compound *AND* inequality

$-14<2x<2$ Subtract 6 from all locations

$-7<x<1$ Divide all locations by 2

PRACTICE SET 13

Solve.

1. $|x+3|>9$

2. $3|y+9|\leq 27$

3. $\frac{1}{4}|y-3|+2<1$

4. $-2|x+4|<22$

5. $|-2x+1|\geq 2$

6. $\left|\frac{x+5}{3}\right|-3>6$

Algebra 2 Summer Review Packet
Student Answer Key

A. Simplifying Polynomial Expressions

PRACTICE SET 1

1. $24x + 3y$
2. $-15y^2 + 37y + 22$
3. $9n - 3$
4. $-22b + 6$
5. $160qx + 110q$
6. $-5x + 6$
7. $74z - 24w$
8. $56c - 117$
9. $-27x^2 + 54x - 9$
10. $-y + 31x + 42$

B. Solving Equations

PRACTICE SET 2

1. $x = 7$
2. $x = 26$
3. $x = 9.5$
4. $x = 13$
5. $x = 3.5$
6. $x = 16$
7. $x = 19$
8. $x = -2.8$
9. $x = 9.5$
10. $x = 0$

PRACTICE SET 3

1. $V = W - Y$
2. $w = \frac{9}{r}$
3. $f = \frac{9 - 2d}{-3} = -3 + \frac{2}{3}d$
4. $x = \frac{10 - t}{d} = \frac{10}{d} - \frac{t}{d}$
5. $g = \frac{P + 1620}{180} = \frac{P}{180} + 9$
6. $x = \frac{9y + u + 5h}{4}$

C. Rules of Exponents

PRACTICE SET 4

1. c^8
2. m^{12}
3. k^{20}
4. 1
5. $p^{11}q^7$
6. $9z^9$
7. $-t^{21}$
8. $3f^3$
9. $60h^8k^5$
10. $\frac{a^3b^4}{3c}$
11. $81m^8n^4$
12. 1
13. $30a^3b^4c$
14. $4x$
15. $24x^4y^7$

D. Binomial Multiplication

PRACTICE SET 5

1. $x^2 + x - 90$
2. $x^2 - 5x - 84$
3. $x^2 - 12x + 20$
4. $x^2 + 73x - 648$
5. $8x^2 + 2x - 3$
6. $50 - 100x + 18x^2$
7. $-6x^2 - 20x - 16$
8. $x^2 + 20x + 100$
9. $25 - 10x + x^2$
10. $4x^2 - 12x + 9$

E. Factoring

PRACTICE SET 6

1. $3x(x + 2)$
2. $4ab^2(a - 4b + 2c)$
3. $(x - 5)(x + 5)$
4. $(n + 5)(n + 3)$
5. $(g - 4)(g - 5)$
6. $(d + 7)(d - 4)$
7. $(z - 10)(z + 3)$
8. $(m + 9)^2$
9. $4y(y - 3)(y + 3)$
10. $5(k + 9)(k - 3)$

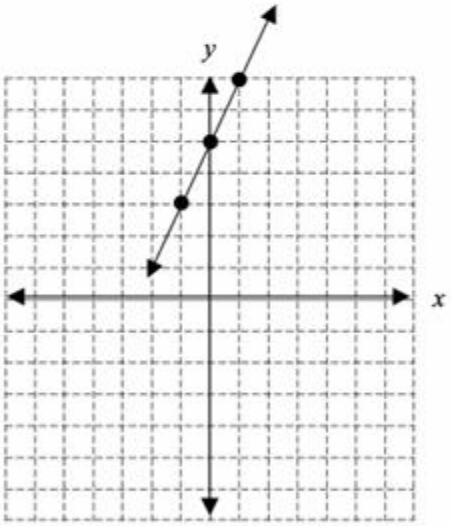
F. Graphing Lines

PRACTICE SET 7

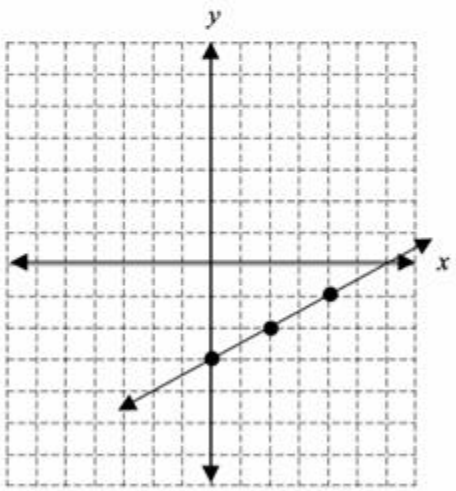
1. **–3**
2. $\frac{2}{3}$
3. $-\frac{1}{2}$
4. **0**
5. **1**
6. **undefined**

PRACTICE SET 8

1. Slope: **2** *y*-intercept: **5**

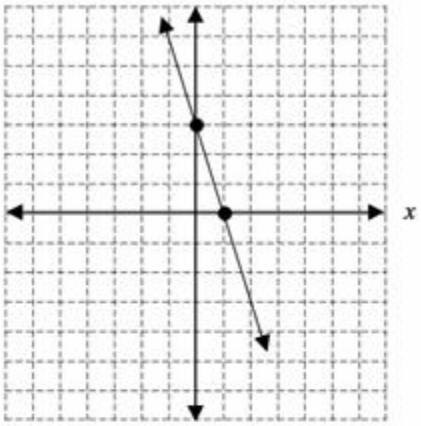


2. Slope: $\frac{1}{2}$ *y*-intercept: **-3**



PRACTICE SET 9

1.



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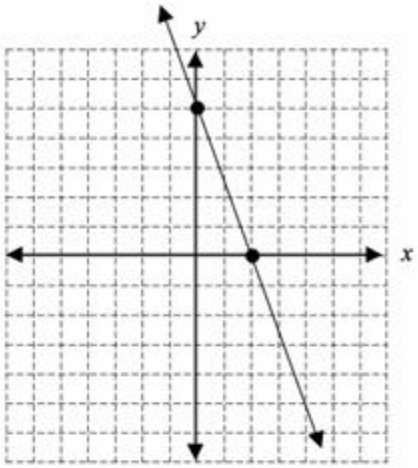

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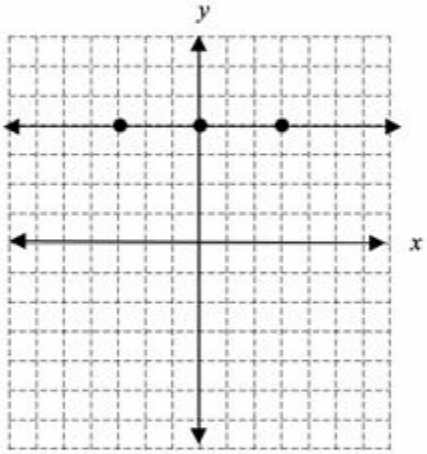

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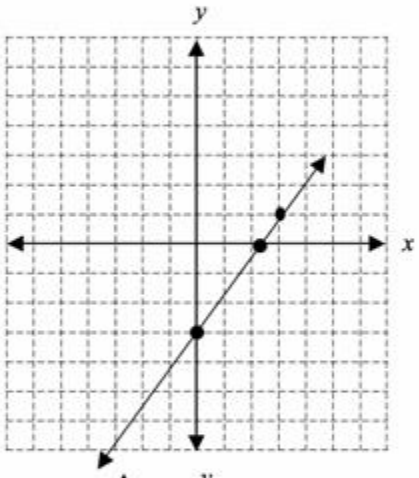

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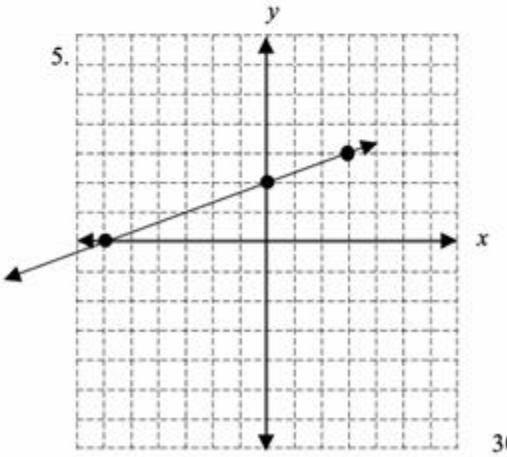

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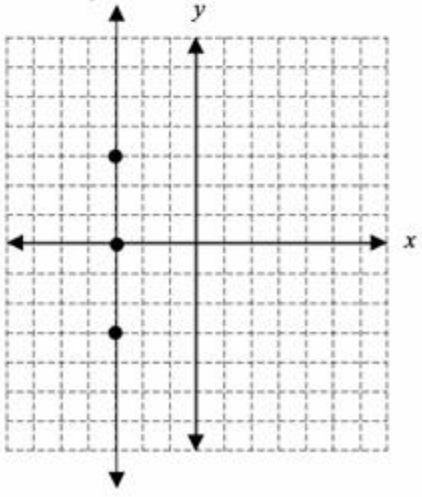

G. Solving Inequalities

PRACTICE SET 10

1. $y \le -6$ 2. $z \ge 6$ 3. $x \ge -48$ 4. No solution

5. All real numbers 6. $2 < AB < 6$

Compound Inequalities

PRACTICE SET 11

1. $-5 < x < 2$ 2. $b < 2 \text{ or } b > 5$ 3. $-1 < x < 8$

4. All real numbers 5. No solution 6. $x < -16 \text{ or } x > \frac{2}{5}$

H. Absolute Value Equations

PRACTICE SET 12

1. $x = 6, x = -6$ 2. $y = 8, y = -4$ 3. No solution

4. $y = -\frac{1}{2}, y = -\frac{5}{2}$ 5. $x = 1$ ($x = -\frac{9}{5}$ is extraneous) 6. No Solution ($x = \frac{1}{2}$ and $x = \frac{13}{8}$ are extraneous)

7. $x = -1, x = -3$

Absolute Value Inequalities

PRACTICE SET 13

1. $x > 6 \text{ or } x < -12$ 2. $-18 \le y \le 0$ 3. No solution

4. All real numbers 5. $x \le -\frac{1}{2} \text{ or } x \ge \frac{3}{2}$ 6. $x > 22 \text{ or } x < -32$